Engineering the Dynamic Range of Si_3N_4 Nonlinear String Resonators

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Summary. Nanomechanical resonators are prone to nonlinear dynamic behavior at forces that are only a few nN. In this work, we introduce a novel design that allows us to engineer the dynamic range of a Si_3N_4 nonlinear string resonator using a pair of compliant supports. The design comprises a suspended string and two support beams that act as the compliant supports and form an H-shaped structure. By changing the support beams' width and length, we are able to tune the state of stress in the string and thus control its quality factor Q and Duffing constant γ. Our novel design allows engineering novel high-Q nonlinear resonators with large dynamic range.

Introduction

Although the Duffing nonlinearity has been frequently observed in nanomechanical systems, the engineering of it by geometric design has received little attention [1]. For studying and utilizing the nonlinear response of nano resonators, it is essential to precisely predict or tune their dynamic range in a simple and robust design. In this work, we decouple the stress-length dependence of high-stress Si_3N_4 nano strings [2] by introducing a pair of support beams. Our H-shaped structure offers new knobs for engineering nonlinearity in string resonators while maintaining dynamic similarity with doubly clamped strings. By changing the widths of support beams, we engineer the pre-stress of the string and thus control its Q factor and Duffing constant γ in a large frequency range.

Experimental procedure

Our samples are fabricated from 340nm thick high-stress Si_3N_4 with initial stress around 1.1GPa, which is deposited by low pressure chemical vapor deposition (LPCVD) on a silicon substrate. The scanning electron microscope (SEM) image of our H-shape structure is shown in Figure 1(a) and our measurement set-up is shown in Figure 1(b). A piezo actuator is used to drive the samples into resonance while a Polytec Laser Doppler Vibrometer is used to measure their out-of-plane deflection, as shown in Figure 1(c). The measurement is performed at room temperature and to minimize the air damping, the air pressure is pumped below $2{\times}10^{-6}$ mbar.

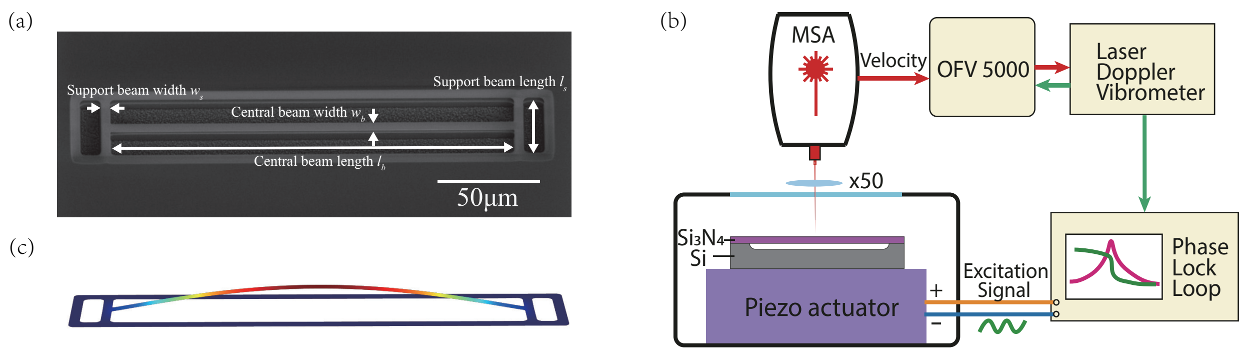

Figure 1: Experimental characterization of the H-shaped resonators. (a) Scanning electron microscope images of an H-beam with l_b=200µm, w_b=4µm, l_s=31µm, w_s=4.5µm. (b) Schematics of measurement set-up. (c) Fundamental mode shape of the H-beam in (a).

Modelling and the governing equation

Motion-induced tension modulation is the dominant source of nonlinearity in nanomechanical systems that gives rise to a cubic spring constant (Duffing term in the equation of motion) as follows [3]:

$$\ddot{x} + Q^{-1}\omega_0\dot{x} + \omega_0^2\left(x + \gamma x^3\right) = f\cos\omega t \tag{1}$$

Here, x is the generalized coordinate associated with the fundamental mode which is shown in Figure 1(c), Q is the mechanical quality factor, ω_0 is the fundamental eigenfrequency, γ is the Duffing coefficient, f is the force per unit mass acting on the nanoresonator of modal mass m and ω is the excitation frequency.
The critical vibration amplitude of the resonator associated with the onset of nonlinearity can be obtained as follows:

$$a_c = \left(\frac{64}{27}\right)^{0.25} \frac{1}{\sqrt{Q\gamma}} \tag{2}$$

We note that Q and γ in nanoresonators are functions of the pre-stress σ. Therefore, by controlling geometrical parameters of the support beams in our design, we can engineer the pre-stress σ in the central string resonator, tune Q and γ, and thus a_c of the resonator.

Control of quality factor Q and Duffing coefficient γ

Quality factor Q

For each resonator, we perform frequency sweeps in linear regime and record the resonant frequency of the fundamental out-of-plane mode by Lorentzian fits, as shown in Figure 2(a). After obtaining the fundamental resonant frequency, we perform ring-down measurements and obtain Q factors, also shown in Figure 2(a). In Figure 2(b), the simulated Q factors and the measurements on the H-beams with the same configuration are plotted against pre-stress σ in the string, which is tuned by the widths of the support beams.

Duffing coefficient γ

In this work, we extract γ by driving the string resonator in the nonlinear regime and sweeping the frequency in the spectral neighborhood of the fundamental resonance as shown in Figure 2c. By performing the experiments for multiple drive levels we find the "backbone" curve of the resonator and extract the Duffing constant γ using the following simple formula:

$$\omega_{backbone}^2 = \omega_0^2 \left(1 + 0.75\gamma x_{max}^2\right) \tag{3}$$

For the design with l_b=200µm and w_b=4µm, we are able to tune Q factor down by 49% and tune γ down by 75% by varying w_s from 6µm to 1µm. With this combined effort, we can tune the critical vibration amplitude a_c of the resonator up to nearly 300% compared to simple doubly clamped string resonators. This shows the potential of the new design in enhancing the dynamic range of high-Q resonators, which is essential for sensing applications.

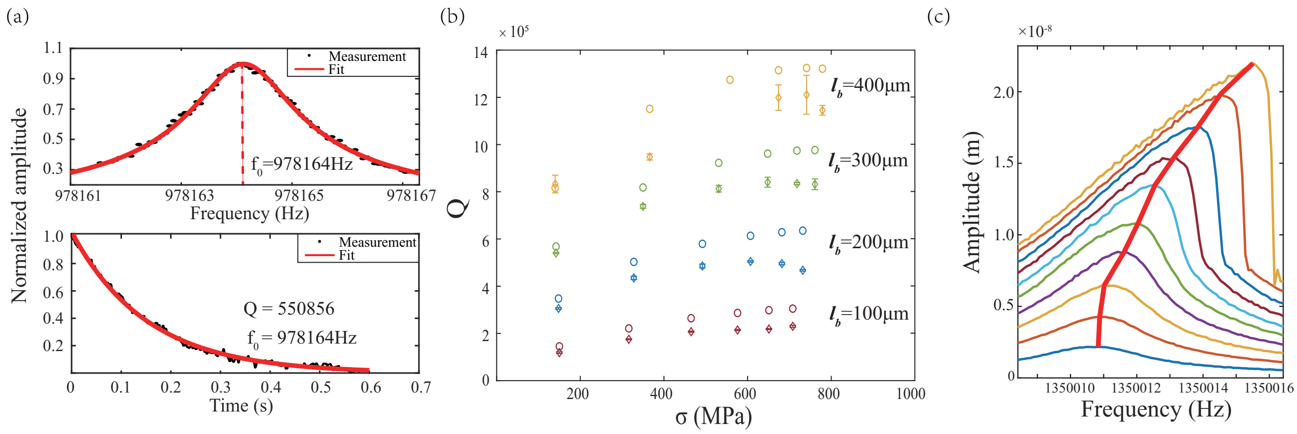

Figure 2: (a) The Lorentzian response of the H-beam showed in the Figure 1(a) excited in its fundamental mode under 1.25×10^{-5}mbar air pressure and the ring-down measurement of it. The y axis is the normalized to the maximum amplitude. (b) Variation of Q factor according to σ for H-beams with w_b=2µm and different l_b. Hollow circles and hollow diamonds with error bars are simulations and experimental results of Q factor respectively. (c) The Duffing effect of a doubly clamped beam with l_b=200µm and w_b=4µm excited by larger forces. The red line is the "backbone" curve.

Conclusions

We propose a new design for enhancing the dynamic range of high-Q Si_3N_4 string resonators by means of compliant supports. By tuning geometrical parameters at the boundaries, we are able to engineer the state of stress in the string from 0.1GPa to 1.0GPa. As a result, we can engineer the Q factor and Duffing constant γ of the resonator , and thus control its dynamic range.

References

[1] S. Dou, B. S. Strachan, S. W. Shaw, J. S. Jensen. (2015) Structural optimization for nonlinear dynamic response. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 373: 20140408.

[2] M. Bückle, Y. S. Klaß, F. B. Nägele, R. Braive, E. M. Weig. (2021) Universal length dependence of tensile stress in nanomechanical string resonators. *Physical Review Applied* , 15: 034063.

[3] I. Kozinsky, H.W. Ch. Postma, O. Kogan, A. Husain, M. L. Roukes. (2007) Basins of Attraction of a Nonlinear Nanomechanical Resonator. *Physical Review Letters* , 99: 207201.