A numerical package for model order reduction of large dimensional finite element systems of nonlinear vibrating structures based on invariant manifold theory

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Summary. Dimensionality reduction through parametrisation of the system motion along a low dimensional invariant-based span of the phase space represents the most efficient technique for deriving reduced order models (ROM) of structures vibrating with large amplitudes. In this work we present the first release of an efficient software for deriving reduced models of structures based on the Direct Parametrisation of Invariant Manifolds (DPIM). The package exploits an algorithmic implementation of the method tailored for mechanical systems, hence achieving low memory consumption and unprecedented speed. Examples of large scale systems of industrial interest are shown and comparisons with experimental data and full order numerical simulations are reported.

Introduction

The computation of periodic orbits in large scale finite element models of vibrating structures represents a major challenge from a numerical standpoint. Indeed, full order simulations have computational times and memory requirements that make numerical integration schemes directly applied to finite element models rarely applicable at an industrial level [1]. As a result, dimensionality reduction techniques need to be applied. In this framework, the lack of invariance of modal subspaces hinder the applicability of linear projection methods as the modal decomposition or the proper orthogonal decomposition [2], since the embedding defined by this class of techniques is not invariant. Furthermore, the identification of coupled modes, that need to be accounted for in order to define a proper ROM, is generally impossible. As a result, nonlinear reduction methods are expected to provide higher accuracy and cheaper computational demands [3]. Among this class of techniques, the Direct Parametrisation Method for Invariant Manifolds [4, 5] allows deriving invariant subspaces together with the dynamics along such sets in an efficient manner. In this work, we report a numerical package that exploits a specific implementation of the method tailored for vibratory systems. Examples of structures of industrial relevance analysed with the proposed method are reported, together with validation against full order numerical simulations.

Method

Nonlinear vibrations of continuous structures discretised by the finite element method and featuring geometric nonlinearities are considered. In this framework, the discretised equations of motion stemming from the FE procedure leads to the following system of differential equations:

$$\begin{aligned} &\mathbf{M}\dot{\mathbf{V}} + \mathbf{C}\mathbf{V} + \mathbf{K}\mathbf{U} + \mathbf{G}(\mathbf{U},\mathbf{U}) + \mathbf{H}(\mathbf{U},\mathbf{U},\mathbf{U}) = \mathbf{F}, \\ &\dot{\mathbf{U}} = \mathbf{V}, \end{aligned} \tag{1}$$

with $\mathbf{M}$ mass matrix, $\mathbf{C}$ damping matrix, $\mathbf{K}$ stiffness matrix, $\mathbf{U}$ nodal displacement, $\mathbf{V}$ nodal velocity, $\mathbf{G}(\mathbf{U},\mathbf{U})$ quadratic nonlinearity tensor, $\mathbf{H}(\mathbf{U},\mathbf{U},\mathbf{U})$ cubic nonlinearity tensor, and $\mathbf{F}$ external forcing vector. $\dot{(\cdot)}$ denotes the time derivative operator. The number of degrees of freedom of Eq. (1) typically between 10^3 and 10^9. Dimensionality reduction is performed using the DPIM. To this aim, we introduce a nonlinear change of coordinates aimed at parametrising the system motion along a low dimensional invariant based-span of the phase space. The mappings introduced to parametrise nodal displacement and velocity are defined as [6]:

$$\mathbf{U} = \boldsymbol{\Psi}(\mathbf{z}), \qquad \mathbf{V} = \boldsymbol{\Upsilon}(\mathbf{z}), \tag{2}$$

where $\mathbf{z}$ are normal coordinates, i.e. the coordinates that describe the system motion along the embedding. The reduced dynamics is given as:

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}). \tag{3}$$

Both mappings $\boldsymbol{\Psi}(\mathbf{z})$, $\boldsymbol{\Upsilon}(\mathbf{z})$ and reduced dynamics $\mathbf{f}(\mathbf{z})$ are obtained upon their substitution into Eq. (1), operation that yields the following invariance equations:

$$\begin{aligned} &\mathbf{M}\nabla_{\mathbf{z}}\boldsymbol{\Upsilon}(\mathbf{z})\mathbf{f}(\mathbf{z}) + \mathbf{C}\boldsymbol{\Upsilon}(\mathbf{z}) + \mathbf{K}\boldsymbol{\Psi}(\mathbf{z}) + \mathbf{G}(\boldsymbol{\Psi}(\mathbf{z}),\boldsymbol{\Psi}(\mathbf{z})) + \mathbf{H}(\boldsymbol{\Psi}(\mathbf{z}),\boldsymbol{\Psi}(\mathbf{z}),\boldsymbol{\Psi}(\mathbf{z})) = \mathbf{0}, \\ &\mathbf{M}\nabla_{\mathbf{z}}\boldsymbol{\Psi}(\mathbf{z})\mathbf{f}(\mathbf{z}) = \mathbf{M}\boldsymbol{\Upsilon}(\mathbf{z}). \end{aligned} \tag{4}$$

The resulting reduced model can then be solved to retrieve the frequency response curve of the associated structure.

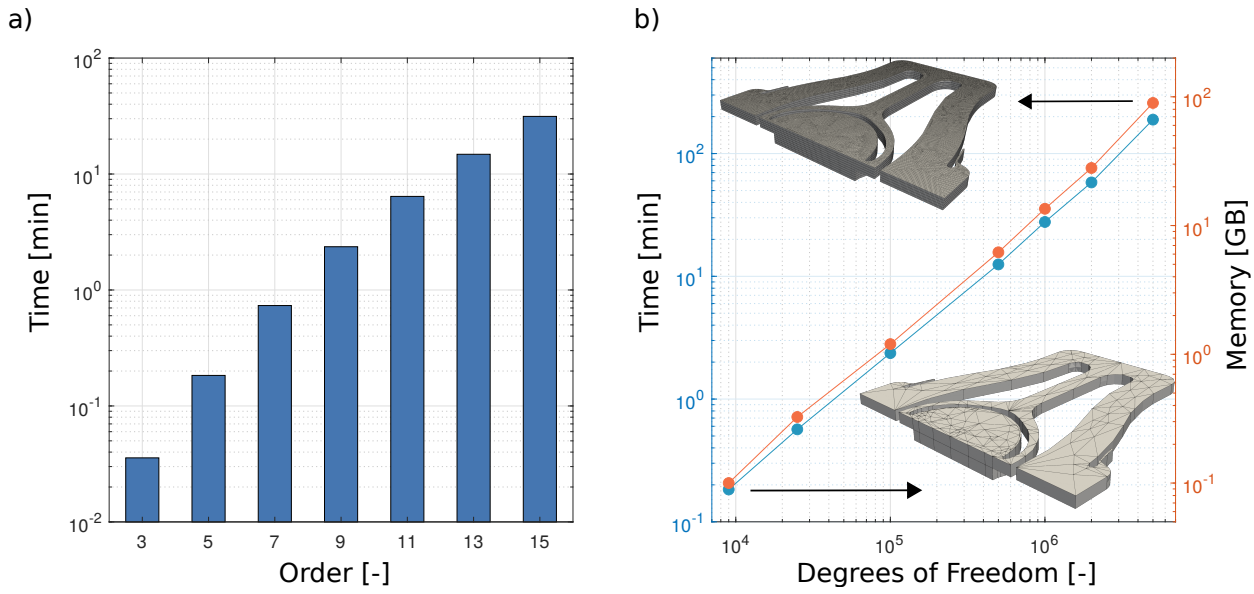

Figure 1: (a) Computing times required to reach a given parametrisation order, for a mesh of 9732 degrees of freedom. (b) Computing times required to perform an order 5 parametrisation for meshes of increasing refinement. All the analyses were performed on a desktop workstation with an AMD®Ryzen 5950X processor and 128GB RAM.

Mappings and reduced dynamics are polynomial functions of the normal coordinates and for each monomial the linear system of the associated homological equation needs to be solved. The DPIM is formulated from the general theory provided by the parametrisation method for invariant manifold, see [7], and has been applied to FE structures in [5] and [4]. In the proposed software, the homological equations are solved at the elementary level for 3D elements, which in turn allows avoiding the computation of the full nonlinearity tensor of the system beforehand, leading to very important savings in memory consumption. Furthermore, we leverage the symmetries of the formulation to avoid computing all monomials associated to the expansion, the latter extensively detailed in [4]. This in turn allows deriving an efficient formulation and yields a technique applicable to large finite element models.

The resulting algorithm has been implemented in a Julia package that provides a mean to identify frequency response curves of structures actuated at resonance with high accuracy and performance. An example of the performance provided by the technique is reported in Fig. 1, where results obtained for a single master mode reduction are highlighted. Fig. 1(a) reports the expected time required to parametrise a system of 9732 degrees of freedom up to an asymptotic development of order 15. Fig. 1(b) details the computational time and memory required to parametrise finite element systems of increasing number of degrees of freedom.

Conclusions

In the present contribution we provide an overview of algorithms and results provided by an open-source model order reduction package. This software aims at providing a tool for fast and accurate analysis of nonlinear structures, hence accelerating the design of structural components. Example applications highlight how the proposed package can play a major role in several fields as the semiconductor industry.

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