Passive control of galloping vibrations by means nonlinear energy sinks

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Summary. The present paper aims to analyze the passive control of a structure subject to aeroelastic galloping by using nonlinear energy sinks (NES). A lumped parameter model is adopted, and a steady approach to the aerodynamic loads is considered. A pure cubic stiffness NES is placed inside the prismatic structure. A mathematical model is established, and the Method of Multiple Scales (MMS) is used to build analytical solutions. Bifurcation diagrams can be drawn with these solutions, which allows characterizing the suppression regimes induced by the absorber. The use of coupled NES seems reasonable to passively control the nonlinear aeroelastic galloping.

Introduction

The galloping phenomenon comprises one of several engineering problems ruled by flow-induced vibrations. This phenomenon is essentially nonlinear and characterized by a self-excited mechanism. Limit cycle oscillations (LCO) take place from a particular flow speed, highlighting the galloping bifurcation. These motions can present very large amplitude, leading to fatigue and failure of structural components [1]. In this way, the present paper aims to investigate the application of NES to passive control galloping vibrations. NES consists of a dynamic vibration absorber with nonlinear characteristics that works according with the Target Energy Transfer (TET) theory [2]. In particular to this work, a pure cubic NES stiffness is considered and placed inside a prismatic body subjected to galloping excitation. A steady and nonlinear approach to the aerodynamic loads is considered [1]. Asymptotic analysis is carried out using MMS. The built analytical solutions allow accessing the amplitude and stability of the system's motion with efficiency. Bifurcation diagrams can be generated with these solutions, and the suppression regimes induced by NES can be characterized.

Mathematical Modeling and Asymptotic Analysis with MMS

The model assumes a rigid square prism with mass m, height h, and is supported by a suspension with linear stiffness k and viscous damper coefficient c. An NES is embedded inside of the structure to promote a passive control effect, which consists of an oscillator with a small mass m_n linked to the prism by a pure cubic spring (k_n), and a linear viscous damper (c_n). The motion of prism and NES masses are accounted for by the $y(t)$ and $y_n(t)$ degrees of freedom in the y-direction. Figure 1 presents an illustration of a system immersed in airflow with velocity U aligned to the x-direction, promoting the motion only in the y-direction.

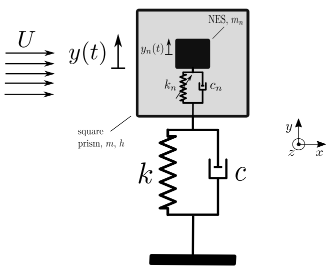


Figure 1: Sprung prism coupled with NES to passive control of the galloping phenomenon.

The governing system's equations of motion are given by:

$$\begin{cases} (1-\epsilon\hat{\mu}_n)\eta''(\tau)+\eta(\tau)=\epsilon\hat{n}\left[\mathcal{A}_1\eta'(\tau)-\mathcal{A}_3\eta'^{\,3}(\tau)\right]-\epsilon\hat{\lambda}_n\upsilon'(t)-\epsilon\hat{\gamma}_n\upsilon^3(\tau) \\ \epsilon\hat{\mu}_n\upsilon''(\tau)+\hat{\lambda}_n\upsilon'(\tau)+\epsilon\hat{\gamma}_n\upsilon^3(\tau)=\epsilon\hat{\mu}_n\eta''(\tau) \end{cases} . \tag{1}$$

with $\tau=\omega_n t$, $()'=d()/d\tau$, $()''=d^2()/d\tau^2$, $\eta(\tau)=y(\tau)/h$, $\omega_n=\sqrt{k/m}$, $\upsilon(\tau)=(y(\tau)-y_n(\tau))/h$, $\mu_n=m_n/m$, $\gamma_n=k_n h^2/(m\omega_n^2)$, $\lambda_n=c_n/(m\omega_n)$, $n=\rho h^2/(2m)$, $V=U/(\omega_n h)$, $\mathcal{A}_1=VC_f^l-2\zeta/n$, $\mathcal{A}_3=C_f^c/V$, $\zeta=c/(2m\omega_n)$, and $\hat{(\,)}=(\,)/\epsilon$.

The MMS assumes the following expansion of the generalized coordinates, $\eta(\tau)=\eta_0(\tau_0,\tau_1)+\epsilon\eta_1(\tau_0,\tau_1)$ and $\upsilon(\tau)=\upsilon_0(\tau_0,\tau_1)+\epsilon\upsilon_1(\tau_0,\tau_1)$ [3]. Replacing this result in Eq. (1), and collecting the common terms with ϵ^0 and ϵ, two set of equations are obtained. The set ϵ^0 comprises only the equivalent linear undamped motion of the prism, and its solution can be written by $\eta_0(\tau_0,\tau_1)=C(\tau_1)e^{j\tau_0}+[c.c.]$, where $C(\tau_1)$ is the prism complex-slowly amplitude, $j=\sqrt{-1}$ is the imaginary unity and $[c.c.]$ refeers to the complex conjugate. Similarly, the NES motion is assumed to be $\upsilon_0(\tau_0,\tau_1)=B(\tau_1)e^{j\tau_0}+[c.c.]$, with $B(\tau_1)$ being the NES complex-slowly amplitude [3]. The ϵ-order problem can be solved by using the previous expressions, and considering $C=(1/2)ae^{j\alpha}$, $B(\tau_1)=(1/2)be^{j\beta}$. Thence, NES motion results in:

$$X_a=\theta_1 X_b+\theta_2 X_b^2+\theta_3 X_b^3 \tag{2}$$

where $X_a = a^2$, $X_b = b^2$, $\theta_1 = 1 + \hat{\lambda}_n^2/\hat{\mu}_n^2$, $\theta_2 = -\frac{3}{2}(\hat{\gamma}_n/\hat{\mu}_n)$, and $\theta_3 = \frac{9}{16}(\hat{\gamma}_n^2/\hat{\mu}_n^2)$. Equation (2) defines the Slow Invariant Manifold (SIM) structure, two folding points can be characterized, and a maximum NES critical damping can be defined by $\lambda_{n_{crit}} = (\sqrt{3}/3)\mu_n$ (i.e., $\lambda_n \leq \lambda_{n_{crit}}$) [3]. Similarly, the equation that describes the prism motion can be combined with Eq. (2), and for the steady state condition the equilibrium points can be calculated by solving the following polynomial equation:

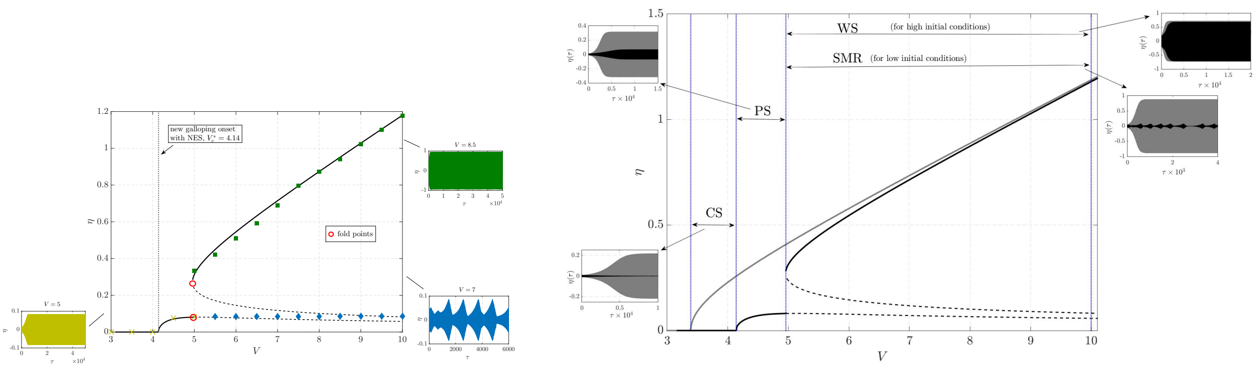
$$X_b^5 + \Gamma_1 X_b^4 + \Gamma_2 X_b^3 + \Gamma_3 X_b^2 + \Gamma_4 X_b + \Gamma_5 = 0, \tag{3}$$

with $\Gamma_1 = \frac{2\theta_2}{\theta_3}$, $\Gamma_2 = \frac{2\theta_1}{\theta_3} + \frac{\theta_2^2}{\theta_3^2}$, $\Gamma_3 = \frac{2\theta_1\theta_2}{\theta_3^2} + \frac{\psi_1}{\psi_2\theta_3}$, $\Gamma_4 = \frac{\theta_1^2}{\theta_3^2} + \frac{\theta_2\psi_1}{\theta_3^2\psi_2}$, $\Gamma_5 = \frac{(\theta_1\psi_1 - 1)}{\psi_2\theta_3^2}$, $\psi_1 = \frac{\hat{n}\mathcal{A}_1}{\hat{\lambda}_n}$ and $\psi_2 = -\frac{3\hat{n}\mathcal{A}_3}{4\hat{\lambda}_n}$. Considering the airspeed with parameter, bifurcations diagrams can be draft based on the solutions of Eqs. (2), (3), allowing to access both amplitude and stability of the motions. These diagrams are essential to characterize the suppression regime responses induced by NES and yours boundaries.

Results and Discussions

The system under study was analyzed considering the following parameters: $n = 4.3\times10^{-4}$, $\zeta = 1.96\times10^{-3}$, $C_f^l = 2.69$, and $C_f^c = 168$ [1]. To validate the analytical approach, time integration results were obtained using the fourth-order Runge-Kutta method with a time step of a 10^{-2} seconds. For each simulated case, an arbitrary initial condition is used to find the steady state motions.

Figure 2(a) compares analytical and numerical bifurcations (♦, ×, ■) of the system considering $\mu_n = 0.05$, $\gamma_n = 1.5$ and $\lambda_n = 0.03\lambda_{n_{crit}}$, where a good agreement between the results is observed. Furthermore, a new bifurcation behavior is induced in the structure by its dynamic interaction with the NES. Unstable branches take place, and news response regimes are detected along the non-linear galloping boundary. The characterization of these suppression regimes is depicted in Figure 2(b). The first one, referred to as CR, comprises a complete suppression of phenomenon. The second regime (PS) occurs when partial suppression is observed through stable LCOs with small amplitudes. The third regime (SMR) is distinguished by competing two different response regimes driven by the initial condition. When small perturbations are imposed, the system exhibits strongly modulated responses (SMR) [2] with lower amplitudes. In contrast, for higher levels, the system jumps to LCOs with larger amplitudes (WS), which results in a weak suppression performance. The upper unstable branch depicts the basin of attraction that delimits these two different behaviors.



(a) ▬ stable with NES, - - - unstable with NES, ♦ SMR, × small amplitude LCO, ■ high amplitude LCO.
(b) ▬ stable with NES, - - - unstable with NES, ▬ without NES.

Figure 2: (a) Validation of MMS-based bifurcation analysis of the system, (b) Characterization of suppression regimes.

Conclusions

The paper investigated the application of an NES with pure cubic stiffness to the passive control of the nonlinear aeroelastic galloping. A benchmark lumped parameter model is used, and the aerodynamic loads are calculated through a nonlinear steady approach. Analytical treatments are carried out based on the MMS, and the solutions are numerically validated. Results comprise a study of the impact of the NES inclusion to the structure and the influence on the nonlinear response in post-galloping with NES. Suppression regimes are characterized based on the bifurcation diagrams in the function of responses induced by NES. To use this type of vibration absorber seems reasonable to control the galloping phenomenon. Further investigation will comprehensively analyze the effect of NES parameters on the boundaries of suppression regimes induced by it and establish the relationship between these regimes and the TET phenomenon.

References

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