Effect of Piezoelectric Coupling on Dynamical Transitions of a Flexible Beam in Viscous Flow

Rajanya Chatterjee*, Chhote Lal Shah†, Sayan Gupta*, and Sunetra Sarkar †

*Department of Applied Mechanics, IIT Madras, India

†Department of Aerospace Engineering, IIT Madras, India

Summary. The effect of the piezoelectric coupling on the dynamical transition of a bimorph cantilever beam in free stream flow at low Reynolds number is numerically investigated in the present study. The results are simulated numerically by using an in-house three-way coupled Immersed Boundary Method (IBM)-based Fluid-structure Interaction (FSI) solver. The effect of piezoelectric coupling at low mass ratios is found to be negligible, as they exhibit periodic dynamics irrespective of the presence of piezoelectric. Without piezoelectric coupling effect, at higher mass ratio of 5.0, the system retains the periodic dynamics. Interestingly, when the piezoelectric coupling effect is introduced, the system transitions from periodic to aperiodic state at the same mass ratio. This study is of importance as it gives insights on the effects of mass ratio at which the efficient energy harvesting of such systems from the piezoelectric material can be possible.

Introduction

The recent advances in the research of alternate energy harvesters, in view of the global depletion of conventional energy resources, is not limited to the large scale solar power or wind energy harvesters. The small scale energy harvesters, powering micro electronic devices to automating small bio-mimetic robots have also been gaining attention from a range of research fields. The energy harvesting strategy from flow induced vibration of flexible flappers, using piezoelectric materials, is one such area, where the system requires a study in fluid structure interaction (FSI), vibration energy harvesters, as well as non linear dynamical characterization of the multiple parameters involved in it. A number of studies in the past and recent times, have contributed towards the development of these flow harvester models, from rigid cylinders with piezo patches in Zhu et al.[1] to more complex bluff body-flexible splitter plate models of Akaydin et al. [2]. In the current study, a flexible cantilever beam model, layered with PZT-5A on both sides (bimorph), has been placed in a viscous fluid, with an oncoming free stream velocity taking the beam to flutter for certain parametric regime. A study on the flutter condition by varying the stiffness and inertia parameters, for an FSI problem was presented in Akcabay et al.[3] and the energy harvesting potential was discussed. However, the authors have not commented on the effect of piezoelectric coupling in the dynamical transition of the system. Therefore, the present study intends to give a comparative analysis on the effect of the presence of piezoelectric coupling on the flutter conditions and energy harvesting capability.

Computational Methodology

A flexible beam layered with PZT-5A on both sides considered in the present study is inextensible where the length of the beam is $L_s = 1.0$. The leading-edge of the beam is fixed and the rest of the body is free to oscillate in the fluid. The non-dimensional governing equation of motion for the flexible beam is given by [3]

$$\beta\frac{\partial^2 \mathbf{X}}{\partial t^2} = \frac{\partial}{\partial s}\left(T_s\frac{\partial \mathbf{X}}{\partial s}\right) - \frac{\partial^2}{\partial s^2}\left(\gamma\frac{\partial^2 \mathbf{X}}{\partial s^2}\right) + \nu V\frac{d}{ds}\left[\delta(s) - \delta(s-L)\right] + \mathbf{F}, \tag{1}$$

where $\mathbf{X} = (X(s,t), Y(s,t))$ is the instantaneous position of the beam, s is the arc length, δ is a Dirac-Delta function, $\beta = \rho_s q/\rho_f L_s$ is the mass ratio (ρ_s, ρ_f and q are the structural density, fluid density and thickness of the beam, respectively). $\rho_f U_\infty^2 L_s$ is used to non-dimensionalize tension coefficient (T_s), $\rho_f U_\infty^2 L_s^3$ is used to non-dimensionalize bending stiffness (γ), t is the non-dimensional time, V is the voltage output non-dimensionalized by $L_s U_\infty(\rho_f/\epsilon)^{1/2}$, ν is the piezoelectric coupling term non-dimensionalized by $L_s U_\infty(\rho_f \epsilon)^{1/2}$ and $\mathbf{F}$ is the Lagrangian forcing acting on the solid body non-dimensionalized by $\rho_f U_\infty^2$. The non-dimensional energy equation for bimorph is given by,

$$\frac{1}{2}\frac{\partial V}{\partial t} + \frac{qq_p}{Rb}V(t) = -\int_0^1 \nu\frac{qq_p(1-q_p)}{2}\frac{\partial^3 \mathbf{x}}{\partial t\partial s^2}, \tag{2}$$

where q_p is ratio of the thickness of one piezoelectric layer with the total thickness of the beam (piezo+substrate), Rb is the non dimensional resistance of the piezoelectric circuit. The structural equation (eq.1) and the energy equation (eq.2) have been discretized by using finite difference method (FDM), details of which can be found in [6]. The viscous flow around the flexible beam is governed by the unsteady Navier-Stokes equations. The momentum conservation and continuity equations in non-dimensional form can be written as,

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla.(\mathbf{uu}) = -\nabla p + \frac{1}{Re}\nabla^2\mathbf{u} + \mathbf{f}; \tag{3}$$

$$\nabla.\mathbf{u} - q_s = 0. \tag{4}$$

Where $\mathbf{u}$ is the flow velocity vector non-dimensionalized by U_∞, $Re = \rho_f U_\infty L/\mu$ is the Reynolds number and pressure p is non-dimensionalized by $\rho_f U_\infty^2$. The momentum forcing term $\mathbf{f}$ is added throughout the solid domain to ensure no slip and no penetration boundary condition is satisfied at the solid boundary and mass conservation is satisfied by adding a mass source/sink term (q_s) to the continuity equation [4]. The fluid, structure and energy equations are coupled in staggered manner where they exchange their information at every time-step.

Results and Discussions

The bimorph flexible beam has been kept in the free stream flow and the simulations are carried out for cases where, $\gamma = 10^{-2}$ and $Re = 200$ are kept constant. The model has been tested with and without piezoelectric coupling for a wide range of β values, and the case where the dynamical transitions due to electrical coupling is tangible, has been presented in this section. At the mass ratio of $\beta \in (0.05, 1.0)$, the system either settles to zero oscillation or shows a periodic flutter, which is unchanged with or without the piezoelectric coupling. However, for mass ratio as high as $\beta = 5.0$, the piezoelectric coupling plays a crucial role in the dynamical transitions. In the absence of piezoeletric coupling, the system exhibits periodic dynamics as shown in Figs. 1(a)-1(c). The tip displacement time history shows no modulation in the amplitude, signifying periodic dynamics (see Fig. 1(a)). The corresponding structural envelope shows mixed mode shape oscillations i.e. 1^{st} and 2^{nd} modes; see Fig. 1(b). The flow field shows the effect of periodic vibration giving rise to a $2p$ periodic vortex street (Fig.1(c)). However, the system transitions to an aperiodic state in the presence of piezoelectric

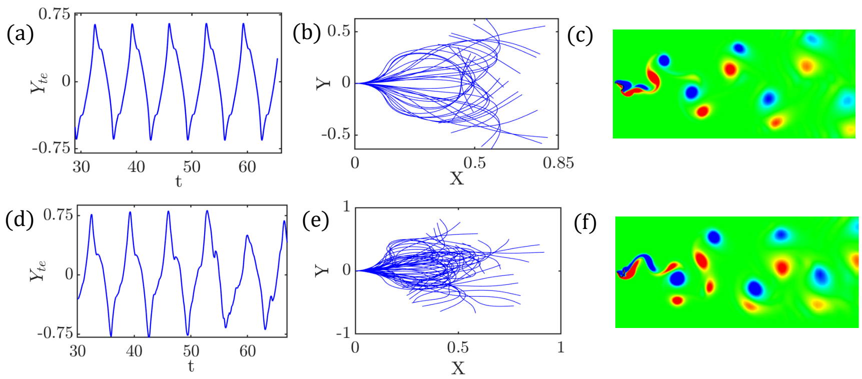

Figure 1: Time histories of beam-tip, beam envelopes and corresponding flowfields without piezo-coupling ((a)-(c)) and with piezo-coupling ((d)-(f)), respectively.

coupling as shown in Figs. 1(d)-1(f) where tip displacement time history shows modulation in the amplitude (Fig. 1(d)) and in the corresponding flow field the vortices are not well-organised (Fig. 1(f)), signifying the characteristics of aperiodic state. The irregular bending is also reflected in the the structural envelope (Fig. 1(e)). The effect of different coupling strengths (ν) are also being investigated currently in our group, to understand the effect of the different piezoelectric materials and will be presented in the the full length paper.

Conclusions

The effect of piezoelectric coupling in the dynamical transition of the three-way coupled FSI system is investigated in the present study. The system evinces periodic dynamics at the higher mass ratio of $\beta = 5.0$ in the absence of piezoelectric coupling. On the contrary, the system transitions to the aperiodic state in the presence of piezoelectric coupling. This study is of importance to identify the proper parametric regime in terms of mass ratio, piezoelectric coupling and bending rigidity in which the energy can be harvested efficiently. The authors are further investigating the effect of different dynamical state on the energy harvesting efficiency of the system.

Acknowledgement

We would like to thank Department of Science and technology (DST), India for funding this work, under the title of "Blade-less Wind Energy Harvesters" and the HPCE IIT Madras for providing with high performance computing facilities.

References

[1] Zhu H.,Zhang C., and Liu W.(2019) Wake-induced vibration of a circular cylinder at a low reynolds number of 100.*Physics of Fluids*.

[2] Akaydin HD. , Elvin N. , and Andreopoulos Y. (2012) The performance of a self-excited fluidic energy harvester. *Materials and Structures*

[3] Akcabay D. T., Young Y. L. (2012), Hydroelastic response and energy harvesting potential offlexible piezoelectric beams in viscous flow, Physics of Fluids 24 054106

[4] Majumdar D., Bose C., Sarkar S. (2020), Capturing the dynamical transitions in the flow-fieldof a flapping foil using immersed boundary method, J. Fluids Struct. 95 102999

[5] Shah, C. L., Majumdar, D., and Sarkar, S. (2020). Delaying the Chaotic Onset in the Flow-Field of Flapping Foil With Flexible Aft Tail. ASME International Mechanical Engineering Congress and Exposition, Vol. 84546, American Society of Mechanical Engineers, 2020, p. V07AT07A027.

[6] Huang W.X., Shin S. J., and Sung H.J. (2007), Simulation of flexible filaments in a uniform flow bythe immersed boundary method.Journal of Computational Physics, 226(2):2206–2228