Hopf Bifurcation in MEMS - (When) Do Such Exist?

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Summary. When considering MEMS/NEMS not just as interesting dynamical systems theoretically, but also for technological applications, a Hopf bifurcation could be a desired property for e.g. increasing sensitivities of a sensor. However, MEMS/NEMS are not known to commonly experience a Hopf point. In this work we investigate a real MEMS system with integrated thermal actuation in passive and active operation modes and investigate this Hopf in MEMS/NEMS question. We show that when the system is operated actively, there are possible parameter ranges which include a Hopf bifurcation.

Background

MEMS are inherently nonlinear, which immediately provides additional properties to be exploited for technological implementation. These advantageous properties are associated with bifurcation points such as saddle-node, transcritical, pitch-fork and Hopf bifurcations. While the former three bifurcations are related to changes of fix-points (or real eigenvalue(s) of the Jacobian) of a system for a selected control parameter, the latter bifurcation is associated with a changing complex-conjugated pair of eigenvalues crossing the complex plane from negative to positive [1, 2]. Furthermore, the former bifurcations have been reported extensively in the literature (for example, the well-known pull-in phenomenon is associated with a saddle-node bifurcation) [3, 4], while the latter is less common, if not rare. For example, in the book by Younis [5] the occurrence of Hopf bifurcations in MEMS is not mentioned. The Hopf bifurcation is related to the damping properties of the system. A single degree-of-freedom (DOF) system (e.g. MEMS/NEMS in their first vibration mode) would therefore require negative damping properties. Furthermore, Gutschmidt and Gottlieb [6] and also Zehnder et al. [4] have observed Hopf (Neimark-Sacker) bifurcations in electrostatically coupled MEMS oscillators when operated near an internal resonance which is also known for parametrically excited systems in general [7] (for micro and macro-scale alike). From other macro-scale examples it is further known that other coupling can also introduce negative damping properties, especially when gyroscopic effects in addition to damping are present in a multi-DOF system. However, for more common MEMS geometries and configurations such scenario are rather rare or not existing.

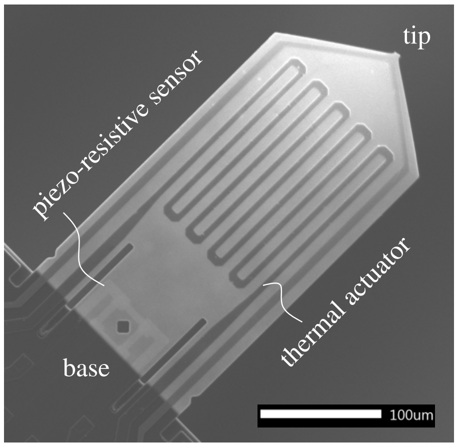

Figure 1: SEM image of the self-actuation, self-sensing cantilever (JEOL JSM-IT300).

In this work we consider such an otherwise typical MEMS (Fig. 1), but with the ability to be operated passively and actively. Unlike other MEMS devices, the considered system has self-sensing and self-actuating capabilities [8] with which a feedback scheme can be introduced and damping properties are altered.

Passive & Active MEMS Model and Dynamics

The motion of the cantilever is generated by the top layer of the composite structure being thermally actuated (see Fig. 1). The deflection of the cantilever tip is monitored by an integrated piezo-resistive sensor located near the supported end (base). Our analysis is based on our previous work [9] in which we derive the governing equations for this composite MEMS system from basic principles. In this work we radically simplify these equations by considering only the first vibration mode of the cantilever and dominating terms are as follows

$$\ddot{q}_w + \delta\dot{q}_w + q_w - \alpha q_\theta = \kappa_{ext} \,, \tag{1a}$$
$$\dot{q}_\theta + \beta q_\theta = \gamma i^2 \,. \tag{1b}$$

q_w and q_θ are the nondimensionalised mechanical and thermal variables of the system representing the modal deflection of the cantilever and the response of temperature difference, respectively. Parameters α, β, γ, δ are classic integration constants originating from a modified Ritz discretization [9], wherein δ is related to the damping. The coupling between the mechanical and thermal systems is determined by the strength of the coupling parameter α. κ_{ext} is an external periodic stimulus. The integrated thermal actuation is modelled as Joule heating, through which the feedback mechanism is introduced with

$$i = i_{DC} + aq_w \,, \tag{2}$$

where i_{DC} is an off-set current with which to control equilibrium states and a is the feedback strength.

Analysis & Results

The analysis emphasizes on the existence of a Hopf bifurcation for equilibrium solutions. However, Hopf (Neimark-Sacker) bifurcations in the presence of external excitation of the system are also of interest. We consider the three following systems: passive (decoupled from the thermal actuation, only Eq. (1a)), thermally coupled but passive Eq. (1), and the actively operated system Eqs. (1) and (2), [10], with their Jacobian matrices accordingly being

$$J_{passive} = \begin{bmatrix} 0 & 1 \\ -1 & -\delta \end{bmatrix}, \qquad J_{coupled} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -\delta & \alpha \\ 0 & 0 & -\beta \end{bmatrix}, \qquad J_{active} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -\delta & \alpha \\ \epsilon & 0 & -\beta \end{bmatrix},$$

with $\epsilon = 2\gamma a i_{DC}$ from substituting (2) into (1b). Investigating the roots λ_i for $i = 1, 2, 3$ of these systems (Fig. 2) reveals the absence and existence of Hopf bifurcations. Figure 2a) depicts the roots of systems $J_{passive}$ and $J_{coupled}$, respectively.

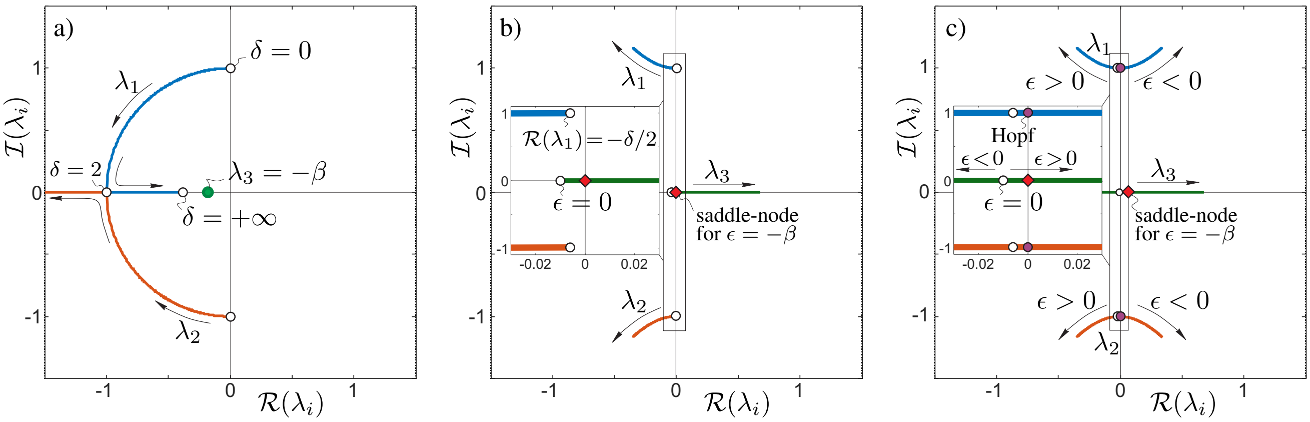

Figure 2: Roots of the MEMS for different operation modes; a) passive & passive coupled systems with all positive system parameters; b) active system with positive system and feedback parameters; c) active system with positive system and negative feedback parameters.

Note, the additional third root for system $J_{coupled}$ (green marker in Fig. 2a)). For positive values of δ (damping) there are no Hopf bifurcations present in either passive or coupled systems, as expected and observed in literature. Figure 2b) portraits the roots of the active system J_{active} (with feedback) for all system and feedback parameters having positive values. We observe a saddle-node bifurcation [10] but no Hopf! Only for negative values of the feedback parameter ϵ, a Hopf bifurcation is present which can be introduced by a negative off-set current i_{DC} or negative feedback strength a (see Eq. (2)).

Conclusions

In this work we investigate the existence of a Hopf bifurcation for equilibria of an ordinary MEMS structure as typically found in modern technological applications. The analysis includes passive and active operation modes, and reveals the existence of Hopf points for only the active operation mode and when feedback parameters include also the negative range. Although the emphasize is laid on Hopf bifurcations related to equilibria, the presentation will also include investigations of the system subject to external stimuli and the existence of Neimark-Sacker bifurcations.

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