A Reduced Order Model for Steady State Response of Joint Assemblies by Hyper-Reduction and Model-Driven Sampling

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Summary. The dynamic behavior of jointed assemblies exhibiting friction nonlinearities features amplitude-dependent dissipation and stiffness. To develop numerical simulations for predictive and design purposes, macro-scale High Fidelity Models (HFMs) of the contact interfaces are required. However, the high computational cost of such HFMs impedes the feasibility and efficiency of the simulations. To this end, we propose a model-driven method for constructing hyper-reduced order models of such assemblies. Focusing on steady-state analysis, we use the Multi-Harmonic Balance Method (MHBM) to formulate the equations of motion in frequency domain. Next, the reduction basis is constructed through solving a set of vibration problems corresponding to fictitious interface conditions. Subsequently, a Galerkin projection reduces the order of the model. Nonetheless, the necessary fineness of the mesh of nonlinear elements on contact interfaces represents a bottleneck for achieving high speedups. Thus, we implement an adapted Energy Conserving Weighing and Sampling (ECSW) technique for Hyper Reduction (HR) for joint problems, thereby allowing significant speedups for meshes of arbitrary fineness. This feature is particularly advantageous since analysts typically encounter a trade-off between accuracy and computational cost when deciding on the mesh size, whose estimation is particularly challenging for problems of this type. Finally, the accuracy and efficiency of the method are demonstrated through a case study.

Introduction

Friction along the interfaces of jointed assemblies, which are commonly found in mechanical and aerospace engineering applications, results in a significant dissipation of energy under dynamic loading. One contact model used to model friction contact in HFMs consists of a Jenkins element with a unilateral spring in the normal direction [1], assigned to each mating node pair on the contact interface. This formulation is capable of reproducing states of sticking, slipping, and separation, locally on the interface.
Projection-based ROM techniques reduce the size of the dynamical system by projecting it on a suitable low-dimensional subspace, thus providing accurate and efficient solutions. Recently, Gastaldi et al. [2] presented the Jacobian-Projection (JP) method, where the reduction basis is constructed in the frequency-domain in a multi-harmonic context, taking into account the harmonic coupling induced by the nonlinear forces. Additionally, we augment the JP basis using vectors representing forced responses of linear systems, which are essential for a high accuracy on forces. Since the evaluation of the non-smooth nonlinear forces across the interface impedes significant speedups, we employ an energy-conserving sampling and weighing (ECSW) hyper reduction strategy [3], adapted to the MHBM context of our problem, with training that does not require any HF simulations.

Method

Problem Formulation

For a Finite Element (FE) discretization of the mechanical system, we assume the equation of motion to be written in the form

$$\boldsymbol{M}\ddot{\mathbf{u}} + \boldsymbol{C}\dot{\mathbf{u}} + \boldsymbol{K}\mathbf{u} + \mathbf{f}(\mathbf{u}) = \mathbf{p}_{\text{ext}}(t), \qquad \mathbf{p}_{\text{ext}}(t) = \mathbf{p}_0 + \mathbf{p}_{\text{E}}(\Omega t), \tag{1}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}$ is the damping matrix, $\boldsymbol{K}$ is the stiffness matrix, $\mathbf{f}(\mathbf{u})$ is the vector of nonlinear forces, $\mathbf{p}_0$ is a vector representing the static loads (e.g. preclamp forces), and $\mathbf{p}_{\text{E}}(\Omega t)$ is the periodic force acting on the system with a time period $T = 2\pi/\Omega$. We use the Multi-Harmonic Balance Method (MHBM) to formulate our Ansatz of the displacements steady-state response in the frequency domain as

$$\mathbf{u}_{\text{h}}(\text{t}) = \mathbf{U}_0 + \sum_{\text{j}=1}^{\text{H}} \left(\mathbf{U}_{\text{j}}^{\text{c}} \cos(\text{j}\Omega t) + \mathbf{U}_{\text{j}}^{\text{s}} \sin(\text{j}\Omega t) \right) \tag{2}$$

where $\mathbf{U}_0$ collects the coefficients of the 0-th harmonic, while $\mathbf{U}_{\text{j}}^{\text{c/s}}$ represents the cosine/sine components of the j-th harmonic.

Augmented JP Method

We extend the JP projection method proposed in [2] by not only considering the free vibration modes arising from linearization at different fictitious contact forces, but also including the corresponding linearized forced response in the basis. The fictitious contact forces (and related Jacobians) are obtained by imposing the linear steady-state solution at different scaling factors. For each scaling factor, we thereby solve an eigenvalue problem, and a linear forced response problem:

$$(\boldsymbol{J}^{\text{k}} - \lambda_{\text{i}}^{\text{k}}\overline{\boldsymbol{M}})\phi_{\text{i}}^{\text{k}} = \mathbf{0}, \qquad \boldsymbol{Z}^{\text{k}}\mathbf{U}_{\text{lin}}^{\text{k}} = \mathbf{P}_{\text{ext}} \tag{3}$$

where $\boldsymbol{J}^{\mathrm{k}}$ is the multi-harmonic stiffness of the structure incorporating the Jacobian of the nonlinear forces corresponding to the k-th system, $\overline{\boldsymbol{M}}$ is the multi-harmonic mass matrix, $\lambda_{\mathrm{i}}^{\mathrm{k}}$ is an eigenvalue, $\phi_{\mathrm{i}}^{\mathrm{k}}$ is the corresponding eigenvector, $\boldsymbol{Z}^{\mathrm{k}}$ is the dynamic stiffness matrix involving the multi-harmonic mass, damping, and stiffness matrices, and $\mathbf{U}_{\mathrm{lin}}^{\mathrm{k}}$ is the linear dynamic response of the k-th system. After computing, and normalizing the vectors ϕ and $\mathbf{U}_{\mathrm{lin}}$, the harmonic components of the vectors are partitioned. Finally, a Singular Value Decomposition (SVD) procedure is applied to the collected vectors to form a well-conditioned basis for each harmonic component. The steady-state solution is thus approximated by

$$\mathbf{u}_{\mathrm{h}}(\mathrm{t}) = \mathbf{U}_0 + \sum_{\mathrm{j}=1}^{\mathrm{H}} \left(\mathbf{U}_{\mathrm{j}}^{\mathrm{c}} \cos(\mathrm{j}\Omega\mathrm{t}) + \mathbf{U}_{\mathrm{j}}^{\mathrm{s}} \sin(\mathrm{j}\Omega\mathrm{t})\right) \approx \boldsymbol{V}_0 \mathbf{Q}_0 + \sum_{\mathrm{j}=1}^{\mathrm{H}} \left(\boldsymbol{V}_{\mathrm{j}}^{\mathrm{c}} \mathbf{Q}_{\mathrm{j}}^{\mathrm{c}} \cos(\mathrm{j}\Omega\mathrm{t}) + \boldsymbol{V}_{\mathrm{j}}^{\mathrm{s}} \mathbf{Q}_{\mathrm{j}}^{\mathrm{s}} \sin(\mathrm{j}\Omega\mathrm{t})\right). \tag{4}$$

where $\boldsymbol{V}_0$, $\mathbf{Q}_0$ are the reduced basis and the reduced coordinates for the 0-th harmonic component, and $\boldsymbol{V}_{\mathrm{j}}^{\mathrm{c/s}}$, $\mathbf{Q}_{\mathrm{j}}^{\mathrm{c/s}}$ are the reduced basis and reduced coordinates of the cosine/sine components of the j-th harmonic, respectively. Next, we perform a Galerkin-projection of the forces of the system to obtain the reduced system

$$\tilde{\boldsymbol{Z}}\mathbf{Q} + \tilde{\mathbf{F}}(\boldsymbol{W}\mathbf{Q}) = \tilde{\mathbf{P}}_{\mathrm{ext}}, \tag{5}$$

where $\tilde{\boldsymbol{Z}}$ is the reduced dynamic stiffness matrix, $\mathbf{Q}$ is a vector collecting the reduced degrees of freedom of the system, $\boldsymbol{W}$ is the block-diagonal reduction basis, $\tilde{\mathbf{F}}(\boldsymbol{W}\mathbf{Q})$ is the projected nonlinear force vector, and $\tilde{\mathbf{P}}_{\mathrm{ext}}$ is the projected external force vector.

ECSW Hyper Reduction

The idea of the ECSW method is to approximate the nonlinear force vector through attributing weights only to a subset of the nonlinear elements of the mesh in such a way that it approximates an energy-like quantity within a specified tolerance

$$\tilde{\mathbf{F}} = \sum_{e=1}^{\mathrm{n_e}} \boldsymbol{W}_{\mathrm{e}}^{\mathrm{T}} \mathbf{F}_{\mathrm{e}}(\boldsymbol{W}_{\mathrm{e}}\mathbf{Q}) \approx \sum_{e\in E} \xi_{\mathrm{e}} \boldsymbol{W}_{\mathrm{e}}^{\mathrm{T}} \mathbf{F}_{\mathrm{e}}(\boldsymbol{W}_{\mathrm{e}}\mathbf{Q}) = \tilde{\mathbf{F}}_{\mathrm{HR}} \tag{6}$$

where $\mathrm{n_e}$ is the number of nonlinear elements, and E represents only a subset of elements for which the weights ξ_{e} are computed such that the following inequality is satisfied with the least amount of non-zero elements:

$$\|\boldsymbol{G}\xi - \mathbf{b}\|_2 \leq \tau \|\mathbf{b}\|_2, \tag{7}$$

where $\boldsymbol{G}$ is a matrix that stores the element-wise contributions to the training snapshots, which in our case are snapshots of the nonlinear forces readily available from the Alternating-Frequency Time (AFT) scheme involved in the construction of the reduced basis. The entries of vector $\mathbf{b}$ represent the assembly of the energy-like quantity from all the elements for the different training snapshots, and τ is set tolerance.

Numerical Results

We apply the proposed method to study the frequency response (FR) of a forced jointed beam. A sketch of the structure is shown in the top left portion of Fig. 1. The Ansatz consists of 5 harmonics. The plot in Fig. 1 features the FR curves for 4 different amplitudes driving the structure at a frequency close to the first bending mode. As demonstrated in the figure, the HR model reproduces the HF results with high accuracy.

The model whose response is shown Fig. 1 has a mesh of 121 nonlinear elements along the contact interface. This model is denoted by Model (1). Table 1 shows that speedups for Model (1) ranging from 5.5 to 7, demonstrating the efficiency of the method. Another model, Model (2), was created to test the convergence of the model with respect to the number of nonlinear elements. This latter has a mesh of 241 nonlinear elements. The FR curves obtained were identical to those shown in Fig.1. This made it possible to conclude that the coarser mesh was sufficient. The speedups of Model (2) are also shown in table 1. They now range from 9.5 to 42.8. It can be noted that also that the HR wall-clock time for Model (2) is only marginally higher than that of Model (1), thanks to the limited number of elements picked by ECSW. In other words, the proposed procedure could alleviate tedious mesh convergence studies, as one could efficiently reduce larger than optimal models.

Conclusion

We presented a hyper-reduced order modelling method for analyzing the steady-state frequency response of jointed structures. The accuracy of the method was shown to be satisfactory for the case studied, and the associated speedups have been presented. A particularly advantageous feature of the proposed method is that the speedups improve as the HF mesh increases in size, thanks to the sparsity of the hyper-reduction scheme.

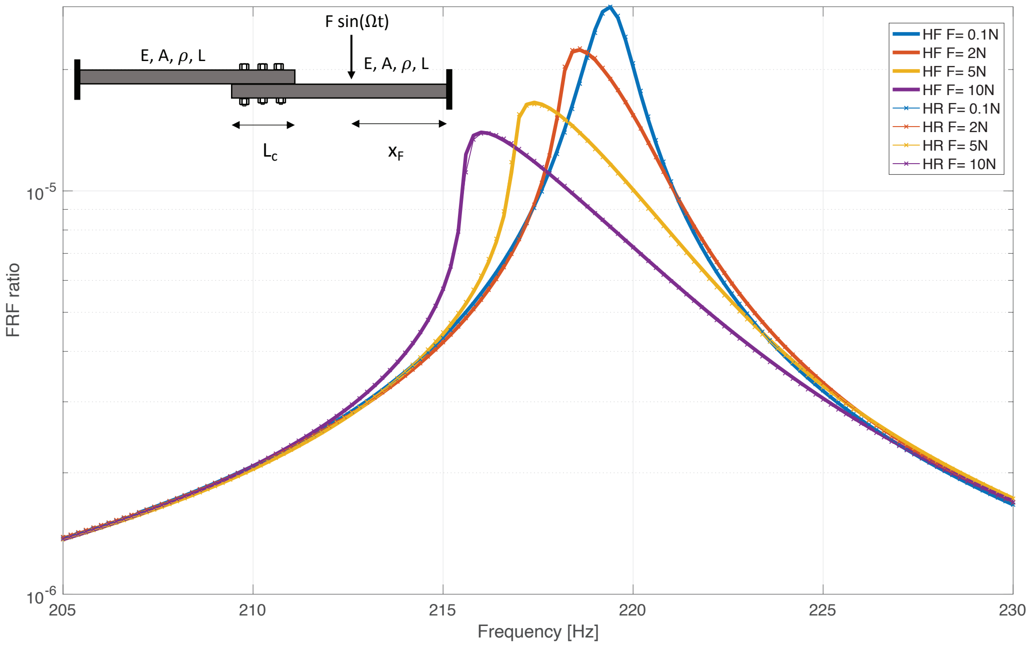

Figure 1: A sketch of the structure is shown in the top left. The plot shows the FR curves for F = 0.1N, 2N, 5N, 10N for the 1st bending mode of the structure using 5 harmonics. The results of the HFM and the HR ROM are denoted by HF and HR, along with the relative forcing. The properties of the structure are: E = 189 GPa, A = 6.25e-4 m^2, L = 0.42 m, ρ = 7820 Kg/ m^3, x_F = 0.24 m, L_c = 0.12 m, bolt load = 1.25 KN, μ = 0.4, k_t = 7.5e9 N/m, k_n = 10e10 N/m

	Mesh (1)			Mesh (2)			
Force [N]	HR [s]	HF [s]	Speedups	HR [s]	HF[s]	Speedups	Ratio of Speedups
0.1 N	29.5	206.4	7.0	57.7	550.7	9.5	1.4
2 N	68.7	404.2	5.9	100.9	1864	18.5	3.1
5 N	85.7	473.5	5.5	115.7	3217.6	27.8	5.0
10 N	95.1	520.4	5.5	129.7	5547.8	42.8	7.8

Table 1: Online computational cost of constructing the FRF curves and associated speedups. Model (1) and Model (2) refer to meshes with 121 and 241 nonlinear elements at the contact interface, respectively. The costs of the bases constructions are respectively 28.4s for Mesh (1) and 97.5s for Mesh(2). The total HR training time for each of the models is less than 1s.

References

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