Nonlinear Damping in MEMS Bridge Resonators

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Summary. This study investigates the nonlinear damping in in-plane micromachined electromechanical resonators theoretically and experimentally. More specifically, precise experiments are performed on an electrically actuated micromachined bridge resonator and the response of the system in the primary resonance region is captured at various AC voltage levels. For the theoretical part, a nonlinear Euler-Bernoulli beam theory is utilised taking into account the geometric, electrostatic, and damping nonlinearities. A general cubic damping model and a Kelvin-Voigt model are considered in the theoretical model. A high-dimensional discretisation is carried out by retaining 10 modes in the Galerkin discretisation. Extensive comparisons are conducted between the experimental data and the theoretical results at different AC voltage levels. It is shown that the resonator exhibits strong nonlinear damping. Different damping models are compared to examine the performance of each model in capturing the damping nonlinearity.

Introduction

Nonlinearly driven micro-electromechanical (MEMS) resonators have drawn increasing attention in the recent decades by wide-spreading potential applications, such as frequency stabilization, filtering, and sensing [1]. Nonlinear vibration of these movable structures has been intensely investigated theoretically and experimentally, chiefly for the electrostatically driven MEMS resonators. A key property affecting the nonlinear vibration of MEMS resonators is damping, in which a variety of physical mechanisms could contribute (e.g., thermoelastic, squeeze film damping, nonlinear mode coupling, and linear viscous damping) [2,3]. As driven the system nonlinearly, the energy dissipation tends to deviate from the linear viscous dissipation and start to vary nonlinearly. Despite the deep study of the nonlinear energy dissipation (or nonlinear damping) in the recent decades [4,5], there is still no comprehensive study on nonlinear damping in MEMS resonators electrostatically driven with relatively large gaps. The present study thoroughly investigates the nonlinear damping in micromachined electromechanical resonators using carefully conducted experiments and an accurate nonlinear theoretical model.

Experimental set-up and theoretical model

In this study, a clamped-clamped in-plane microbeam made of silicon is considered, which is of length L, width b, cross-sectional area A, second moment of area I, Young's modulus E, mass density ρ, and material damping coefficient η,. The microbeam is excited by a stationary electrode separated from the microbeam with a large transduction gap, Fig.1(a). To minimize the effect of squeeze film damping, the microbeam is placed in a vacuum chamber with a pressure of 950mTorr set throughout the experiments. Hence the squeeze film damping effect will be neglected in the theoretical study. The nonlinear equation of motion for the MEMS resonator under consideration is derived using the Euler-Bernoulli beam theory and utilising a Kelvin-Voigt model, resulting in a nonlinear damping mechanism. Apart from the Kelvin-Voigt damping, a general nonlinear cubic damping mechanism (for the transverse motion w) is modelled as well. The equation of motion can be derived as

$$\rho A\partial_{tt}w + c_1\partial_t w + c_2\partial_t w|\partial_t w| + c_3(\partial_t w)^3 + EI\partial_{xxxx}w + \eta I\partial_{txxxx}w - \frac{EA}{2L}\partial_{xx}w\int_0^L(\partial_x w)^2\,\mathrm{d}\,x$$
$$-\frac{\eta A}{2L}\partial_{xx}w\int_0^L 2\partial_x w\partial_{tx}w\,\mathrm{d}\,x - \frac{\varepsilon b[V_{\mathrm{DC}}+V_{\mathrm{AC}}\cos(\omega t)]^2}{2(d-w)^2} = 0, \tag{8}$$

in which c_1, c_2, and c_3 are damping coefficients and d is the MEMS gap width.

The equation of motion is first recast into nondimensional form and then discretised into a set of nonlinearly coupled ordinary differential equations via use of Galerkin technique, considering 10 degrees of freedom. The resultant set is solved using a continuation technique.

Results and Discussion

In this section, the nonlinear resonance response of the MEMS resonator is examined in detail and thorough comparisons are conducted between the theoretical predictions and experimental data. Different linear and nonlinear damping models are examined to determine which model better predict the dissipation in the system. All models are calibrated once and then the damping coefficients are kept fixed as the AC excitation level is increased. Four cases are examined, namely the linear viscous damping model, Kelvin-Voigt damping, cubic nonlinear model (c_1 and c_3) and general quadratic-cubic model. In the results shown in Figs.1(b)-(e), ω_1 denotes the primary nondimensional natural frequency of the microbeam, Ω stands for the nondimensional frequency of excitation, and w_d indicates the nondimensional midpoint transverse oscillation amplitude (with respect to d), measured from the deflected configuration. In all the theoretical results, solid and dashed lines denote stable and unstable periodic solutions, respectively.

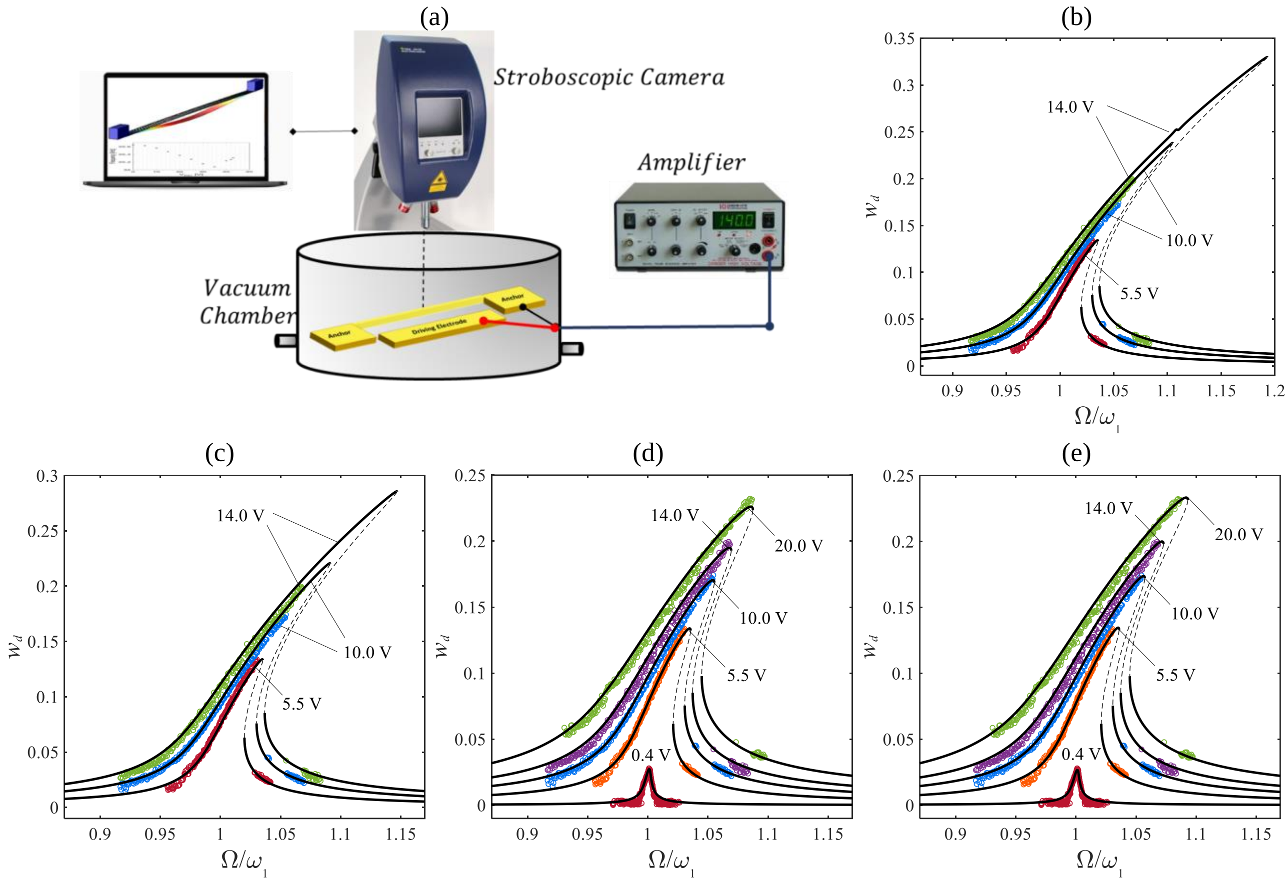

Figure 1: (a) Experimental setup with 3D schematic of the microbeam with length 800 µm, thickness 2.9 µm, width 25 µm, and transduction gap 8 µm. (b, c) Frequency responses of the MEMS resonator at $V_{\mathrm{DC}} = 5$ V and three AC voltages (shown on the curves) based on linear and Kelvin-Voigt damping models, respectively. (d, e) Frequency responses of the MEMS resonator at $V_{\mathrm{DC}} = 5$ V and various AC voltages (denoted on the curves), based on cubic and general quadratic-cubic models, respectively.

Figs.1(b) and (c) correspond to linear and Kelvin-Voigt damping, respectively. For both cases, the frequency response for $V_{\mathrm{AC}} = 5.5$ V is used to calibrate the damping coefficient which is then kept fixed as the AC voltage is increased to larger magnitudes. As seen, both damping models overestimate peak oscillation amplitudes for both AC excitation levels. The results for the cubic damping model are shown in Fig.1(d); for this damping model, the coefficients c_1 and c_3 are calibrated using the two lower-amplitude frequency responses, i.e. those for 0.4 and 5.5 V. The damping coefficients are then kept fixed and the frequency responses at other AC voltages are obtained and compared to the experimental data. It is seen that the cubic model captures the nonlinear damping in the system with a good accuracy; however, it slightly underestimates the peak amplitude, and this becomes more visible at higher AC voltage levels. The final model examined is the general quadratic-cubic damping model and the result for this case is shown in Fig.1(e). This model requires three frequency responses for calibration of three damping coefficients. As seen, this model has the best accuracy in predicting the primary resonance response amplitudes at various AC voltages among all damping models examined.

Conclusions

This study conducted a theoretical-experimental investigation on the nonlinear damping in in-plane MEMS resonators. It was shown that both linear and Kelvin-Voigt models overestimate the peak amplitude at higher excitation levels. The general quadratic-cubic damping model showed excellent agreement with experimental data and worked best among different models.

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