Transient deformation of a beam travelling on a moving rough surface

Yury Vetyukov* and Jakob Scheidl*
*Institute of Mechanics and Mechatronics, TU Wien, Vienna, Austria

Summary. In this paper we study the quasi-static deformation of a beam pressed against a moving rough surface by the field of gravity. While the beam is transported in the axial direction together with the travelling foundation, it deforms and slides in the lateral direction because of the misaligned linear bearings at the boundaries of the control domain. Considering small deflections and a geometrically linear beam model, we present a numerical approach to analyse the time evolution of the deformed state of the beam. Analytical solutions are obtained for several specific cases of the boundary conditions and validated against the numerical results.

Introduction

Lateral deformation of axially moving slender structures with frictional contact is usually undesired in technical applications such as rolling mills [1] or transport belts [2, 3]. The highly nonlinear frictional response encountered in these problems induces dynamical behaviour even at slow quasi-static motion, when inertial effects are negligible. Numerical simulation tools, created to investigate the mechanics or to develop a model-based controller design, rely on the mathematical models of moving contact of deformable bodies at non-material kinematic description. Simplified semi-analytical approaches allow, however, to better understand the nature of the arising phenomena and to validate the complicated numerical schemes. Thus, an analytical study of the motion of an endless beam, transported by a moving rough surface across a control domain, has been presented in [4]. The beam is forced to enter the control domain and to leave it through a pair of linear bearings, which are laterally misaligned relative to each other. The analysis shows, that, as long as the misalignment is small and the maximal friction force is sufficiently large, the stationary deformed configuration of the beam becomes self-similar with infinitely many segments of sliding friction in alternating directions.
It should be noted, that the time evolution of the deformation of the beam on a rough foundation because of the bending moment on a free end has been thoroughly analysed earlier in [5], where the appearance of self-similar solutions was demonstrated as well. A similar formulation with thermally induced bending moments and self-similar deformation pattern studied in [6] relates to the cool down of railway rails after hot rolling. Nevertheless, the presently considered moving contact problem with transport conditions is a novel formulation, described by a different mathematical model.

Problem formulation

In the present paper we extend the results of [4] by investigating the transient deformation of the beam owing to a given law of the lateral motion of the bearing at the entry to the control domain. The model problem is depicted in Fig. 1. The linear bearings, which constrain the motion of the beam at the boundaries, are considered as prismatic joints, such that the deflection w takes on given values and the slope vanishes there, $w' = 0$. The joint at the entry moves in lateral direction over time t according to a given law $w_0(t)$. Under the condition of perfect adhesion, the beam would be transported along the axial coordinate x with the velocity v of the travelling foundation and its deformed shape would become

$$w(x,t) = w_0(t - x/v). \tag{1}$$

However, the boundary conditions at the exit prismatic joint and the bending stiffness of the beam trigger sliding, thus creating a system with a non-trivial dynamic behaviour.
In the geometrically linear setting, small lateral deflections do not affect the axial velocity of a particle of the beam, which thus always coincides with the transport speed $\dot{x} = v$. The lateral component of the velocity of a particle

$$\dot{w}(x,t) = \partial_t w + v w' \tag{2}$$

comprises the local (Eulerian) time derivative $\partial_t w$, computed at a given axial position $x = \mathrm{const}$, and a convective term featuring the derivative with the spatial coordinate $w' = \partial_x w$. The relative velocity $\dot{w}$ between the particle and the foundation determines the Coulomb's dry friction force q according to

$$\dot{w} > 0:\ q = -q_0, \quad \dot{w} = 0:\ -q_0 < q < q_0, \quad \dot{w} < 0:\ q = q_0 \tag{3}$$

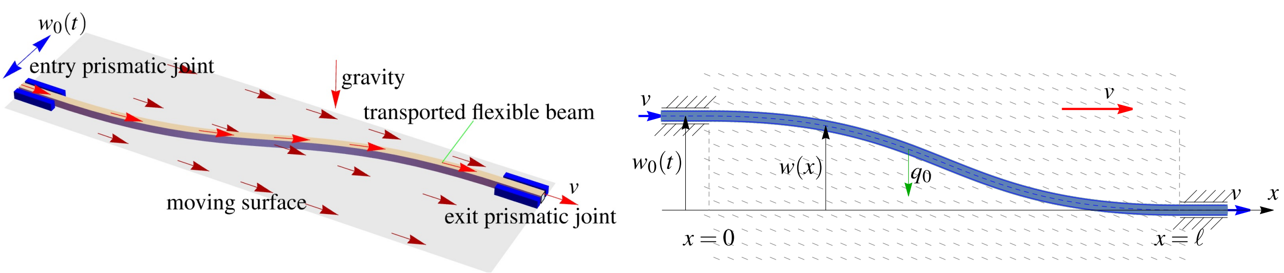

Figure 1: Flexible beam transported across a control domain by a moving rough surface: 3D perspective and view from above

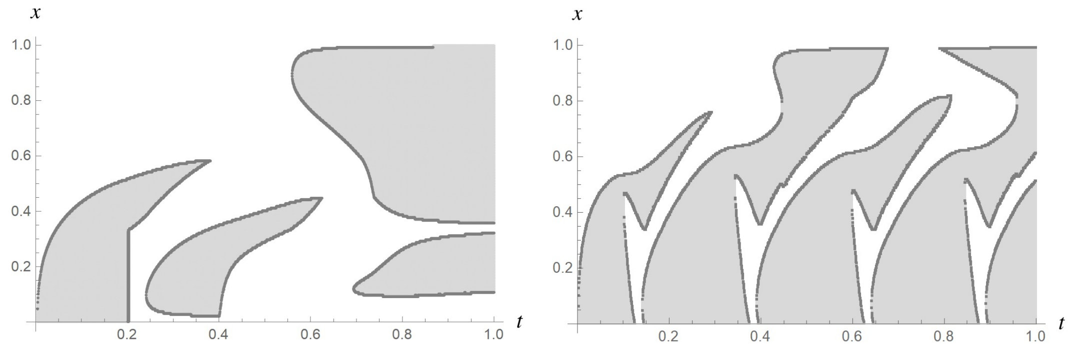

Figure 2: Time evolution of the segments of sliding (gray areas), parameters: $a = 1$, $q_0 = 1$, $v = 1$, unit length of the control domain; left: linear growth with $\bar{w}_0 = 1/300$ and $t_0 = 0.2$; right: harmonic excitation at the entry with $w_0 = \sin(4\pi t)/300$

with q_0 being the sliding friction force, which bounds the static friction force. Now we demand that the beam is in static equilibrium at all times:

$$aw'''' = q \tag{4}$$

with a denoting the bending stiffness of the beam. Complemented with specific boundary conditions, the equations determine the time evolution of the zones of stick and sliding friction as well as the motion of the beam.

Discussion of the solution strategies

The main difficulty in obtaining the solution in terms of $w(x,t)$ and $q(x,t)$ is that the nonlinear equations cannot be resolved for the time derivative $\partial_t w$, which would otherwise facilitate the direct time integration of the evolution law. A regularization with a small inertial term and second-order time derivative would prohibit an analytical solution and require a computationally costly numerical time integration with small time steps. The above outlined quasi-static problem may be tackled numerically in a very efficient manner using the non-material finite element formulation with the transport condition for the deflection field as discussed in [4].
While the stationary solution with $\partial_t w = 0$ in case of constant deflections at the boundaries of the control domain is extensively analysed in [4], the present study focuses on the transient behaviour in response to two distinct cases of the imposed deflection at the entry $w_0(t)$:

- Linear growth followed by constant deflection:

$$w_0 = \begin{cases} \bar{w}_0 t/t_0, & t < t_0 \\ \bar{w}_0, & t \geq t_0 \end{cases} \tag{5}$$

  Sliding is inevitable during the growth stage $t < t_0$, as the boundary condition $w' = 0$ at $x = 0$ contradicts the full adhesion solution (1). One expects, that a "wave" of the length vt_0 shall be transported by the travelling foundation until it reaches the exit prismatic joint. However, a more complicated process with reverse sliding is suggested by numerical analysis at higher values of the amplitude $\bar{w}_0$, see left part of Fig. 2. The stationary solution with alternating segments of sliding friction and a self-similar deformed configuration establishes over time during the subsequent transient stage.

- Harmonic excitation:

$$w_0 = \bar{w}_0 \sin \omega t. \tag{6}$$

  As long as the amplitude $\bar{w}_0$ is small, sliding shall again take place only in the vicinity of the entry point. The length of the segment of sliding shall change in time according to a complicated law, which can approximately be established in an analytical solution. Numerical analysis suggests, however, that higher values of $\bar{w}_0$ result into more segments of sliding near the entry point, see right part of Fig. 2. Finding an estimate for the critical value of the amplitude, at which the solution changes qualitatively, is a challenging mathematical task.

References

[1] Vetyukov, Y., Gruber, P.G., Krommer, M., Gerstmayr, J., Gafur, I. and Winter, G. (2017) Mixed Eulerian–Lagrangian description in materials processing: deformation of a metal sheet in a rolling mill. *Int. J. Numer. Methods Eng.* **109**(10):1371-1390.

[2] Scheidl, J., Vetyukov, Y., Schmidrathner, C., Schulmeister, K. and Proschek, M (2021) Mixed Eulerian-Lagrangian shell model for lateral run-off in a steel belt drive and its experimental validation. *Int. J. Mech. Sci.* **204**:106572.

[3] Schmidrathner, C., Vetyukov, Y. and Scheidl, J. (2021) Non-material finite element rod model for the lateral run-off in a two-pulley belt drive *Z. Angew. Math. Mech.*:e202100135.

[4] Vetyukov, Y. (2021) Endless elastic beam travelling on a moving rough surface with zones of stick and sliding. *Nonlinear Dyn.* **104**:3309-3321.

[5] Stupkiewicz, S., Mróz, Z. (1994) Elastic beam on a rigid frictional foundation under monotonic and cyclic loading. *Int. J. Solids Struct.* **31**(24):3419-3442.

[6] Nikitin, L.V., Fischer, F.D., Oberaigner, E.R., Rammerstorfer, F.G., Seitzberger, M. and Mogilevsky, R.I. (1996) On the frictional behaviour of thermally loaded beams resting on a plane. *Int. J. Mech. Sci.* **38**(11):1219-1229.