Joint Reactions Distribution and Uniqueness in Overactuated Multibody Systems

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Summary. Overactuation is introduced in various mechanisms, especially in parallel kinematic machines. The evident result of overactuation is the non-uniqueness of driving forces required to execute the planned motion. However, in some cases, redundant driving constraints may also affect the uniqueness of joint reactions. In this contribution, the influence of overactuation on the uniqueness of calculated joint reactions is investigated. A procedure for testing the solvability of calculated reactions in the presence of redundant driving constraints is devised. The known numerical methods for constraint reactions uniqueness analysis are adapted to the overactuated case. An example illustrates the investigated problems and demonstrates the proposed analysis methods.

Introduction

Overactuation in multibody systems (MBS) is understood as employing more actuators than required by the number of controlled degrees of freedom of the system. It is introduced due to several reasons, e.g., to eliminate gear backlash or clearances [1], to improve the performance of the system [2], or to reduce torques in joints of the MBS [3]. In the case of parallel kinematics machines, frequently exploited in robotic applications, overactuation may contribute to singularity avoidance [4] or be utilized for obtaining desired torque distribution [5]. Overactuated multibody systems can be treated as a particular case of redundantly constrained systems, where redundancy applies to driving constraints.

Rigid body assumption is commonly utilized in multibody simulations. With this assumption, multibody modeling is less complicated, requires less data, and allows for faster computations. However, in the case of redundantly constrained MBS, this assumption leads to various problems, mainly related to the rank deficiency of the constraint matrix. One of the unwelcome effects is the non-uniqueness of calculated joint reaction forces (or at least some of them). Various methods to handle and analyze overconstrained MBS have been proposed—see, e.g., [6, 7, 8, 9]. Most often, these methods are focused on constraint redundancy introduced by the kinematic structure of the system, whereas overactuation is seldom considered.

In this work, rigid body models of redundantly actuated MBS are analyzed. The research concentrates on the uniqueness of calculated driving forces and joint reaction forces. Methods of uniqueness analysis are developed, and the influence of redundant actuation on joint reaction solvability is pointed out and investigated. An example is provided to exemplify the investigated issues and illustrate the proposed analysis methods.

Outline of the methods and results

Equations of motion of a MBS subjected to geometric and linear nonholonomic constraints can be written as (we assume that absolute coordinates $\mathbf{q}$ are employed to describe the system [10]):

$$\mathbf{M}\ddot{\mathbf{q}} - \mathbf{J}^T\boldsymbol{\lambda} = \mathbf{Q}, \tag{1}$$

where $\mathbf{M}$ is the matrix of inertia, $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers, $\mathbf{Q}$ is the vector of the other generalized forces and velocity-dependent inertial terms.

If no driving constraints are imposed, the constraint matrix is $\mathbf{J}$ defined as:

$$\mathbf{J}^T = \mathbf{J}_K^T = \begin{bmatrix}(\boldsymbol{\Phi}_{\mathbf{q}}^K)^T & (\boldsymbol{\Psi}^K)^T\end{bmatrix}, \tag{2}$$

where $\boldsymbol{\Phi}_{\mathbf{q}}^K$ is the geometric constraints Jacobian and $\boldsymbol{\Psi}^K$ is the linear nonholonomic constraint matrix.

When driving constraints are appended, the constraint matrix becomes:

$$\mathbf{J}^T = \mathbf{J}_{KD}^T = \begin{bmatrix}(\boldsymbol{\Phi}_{\mathbf{q}}^K)^T & (\boldsymbol{\Psi}^K)^T & (\boldsymbol{\Phi}_{\mathbf{q}}^D)^T & (\boldsymbol{\Psi}^D)^T\end{bmatrix}, \tag{3}$$

where superscript D stands for driving constraints.

The presence of redundant constraints makes the constraint matrix $\mathbf{J}$ rank deficient. The number of redundant constraints is indicated by the magnitude of the matrix $\mathbf{J}$ rank deficiency. As a result of redundant constraints existence, at least some of the joint reaction forces are non-unique. There are some methods that make it possible to determine whether or not the reactions in a specified joint are solvable (e.g., [6, 8]). In the present work, these methods were adapted to analyze systems with driving constraints.

To check if the driving constraints influence the uniqueness of calculated joint reactions, one must first analyze the constraint matrix from Eq. (2) that represents the MBS structure independently from its actuation, and then the complete constraint matrix as of Eq. (3). In this paper, we have proven that **if the driving constraints are non-redundant** (i.e., when the magnitude of matrix $\mathbf{J}$ rank deficiency does not change after introducing driving constraints), then **the solvability of joint reactions remains unchanged**; moreover, the driving forces are unique.

On the other hand, **if the system becomes overactuated** (i.e., the magnitude of rank deficiency increases after the introduction of driving constraints), then: (1) previously non-unique joint reactions remain non-unique, (2) **some of otherwise unique joint reactions may become non-unique**, (3) some (or all) driving forces are non-unique. Note that the methods developed in the present work enable us to determine the solvability of any individual joint reaction before and after the introduction of driving constraints. The crucial point is that the results of the uniqueness analysis obtained for joint reactions alone may differ from those obtained for joint reactions together with redundant driving forces.

Example

The developed analysis methods were used to investigate the uniqueness of joint reactions and driving forces in a redundantly constrained and overactuated mechanism with nonholonomic constraints presented in Fig. 1. Three different actuation variants were analyzed: without driving constraints, with non-redundant driving constraints, and with redundant driving constraints. The results of performed analyses corroborated the findings presented in this contribution.

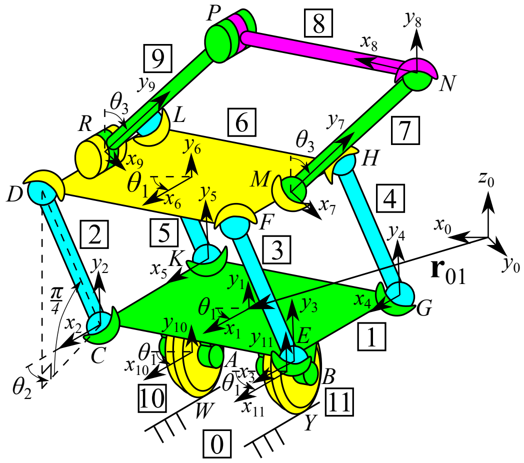

Figure 1: Redundantly constrained and overactuated mechanism with nonholonomic constraints

Conclusions

Overactuation may, in some cases, affect the uniqueness of joint reactions. For the overactuated systems, it is useful to perform a two-stage uniqueness analysis—with and without considering the driving constraints. Such a proceeding enables us to determine the origin of the reaction non-uniqueness of the considered MBS—whether it comes from the system's structure or the redundancy of the driving constraints imposed on the MBS.

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