

## Stability of a driver-vehicle system with steering and throttle control

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**Summary.** The stability of a vehicle's driving state depends on the mechanical properties of the vehicle and the road conditions, but to a large amount also on the characteristics of the human driver. The aim of this talk is to study the influence of the driver's reaction on the stability of the steady cornering motion, which for constant control input is asymptotically stable.

### Introduction

The stability of a vehicle's driving state depends on the mechanical properties of the vehicle and the road conditions, but to a large amount also on the characteristics of the human driver. The human driver might be replaced by a robot in the case of an automated vehicle. The aim of this talk is to study the influence of the driver's reaction on the stability of the steady motion, which for constant control input is asymptotically stable.

We investigate the loss of stability of a controlled understeering vehicle along a steady-state cornering motion, according to the model described in [1]. To control the trajectory of the vehicle, the human driver is assumed to either adjust the front steering angle  $\delta_F$  or the driving torque  $M_R$  of the rear wheels, according to the deviation of a point  $P$  from a reference circle (Fig. 1). If we denote the deviation of the point  $P$  from the reference circle by  $\Delta r_P$  and the deviation of the control input  $u \in \{\delta_F, M_R\}$  from the stationary value by  $\Delta u$ , the "simplified precision model" [2] takes the form

$$T_M \frac{d\Delta u(t)}{dt} + \Delta u(t) = c_P \Delta r_P(t - \tau) + c_D \frac{d\Delta r_P}{dt}(t - \tau), \quad (1)$$

with human reaction time  $\tau$ . Delay time  $T_M$ , the control gains  $c_P$  and  $c_D$  depend on the driver's skills and on the handling behaviour of the vehicle. A similar control loop with delay for a vehicle dynamics was investigated in [3].

While frequently used driver models, based on the above modelling approach, are applied for a linear driving regime, a nonlinear design is focussed here. In particular, the brush tyre model, [4] is used for the nonlinear tyre forces at higher lateral accelerations. The driver model is able not only to control the given trajectory with the steering angle, but also with the longitudinal tyre force resulting from the rear wheel drive. This is a skill of a human (expert) driver, but rarely addressed in human driver modelling, but will become in particular important also at automated driving, when individual drive torques may be used to stabilise critical driving manoeuvres in the nonlinear driving regime.

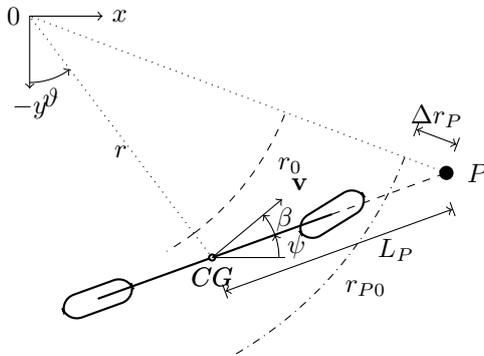


Figure 1: Geometric relations for the driver's preview model: To follow a circle with radius  $r_0$ , the driver spots a point  $P$  at a distance  $L_P$  straight ahead, which should move along a circle with radius  $r_{P0}$ .

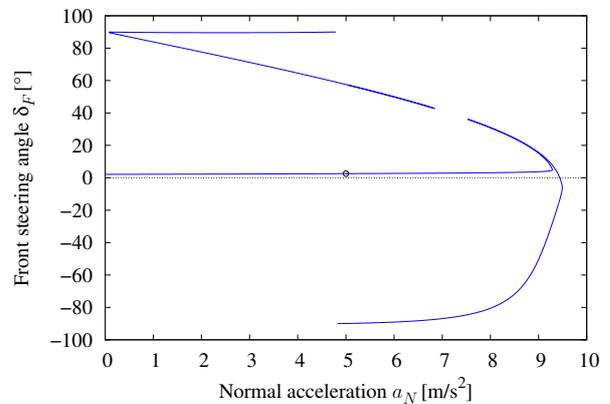


Figure 2: Handling diagram for the considered vehicle for a steady cornering motion along a circle with radius 80 m. For the considered steady velocity  $v = 20$  m/s the equilibrium point is asymptotically stable.

### Preliminary results

In order to investigate the linear stability of the cornering motion, we choose parameters and control inputs, for which the steady state cornering motion is asymptotically stable. As an example for such a stable motion we select the point indicated in Fig. 2, corresponding to a constant speed  $v = 20$  m/s. Neglecting the reaction time  $\tau$  we select control gains, for which the vehicle-driver model is still asymptotically stable. Fixing these parameters, we vary the reaction time  $\tau$  and determine the critical parameter values, for which a loss of stability due to a Hopf bifurcation occurs.

As can be seen from Figs. 3 and 4, the steering control depends very sensitively on the reaction time for the considered parameter values, whereas the critical reaction time  $\tau_c$  for pure throttle control is far beyond the usual values.

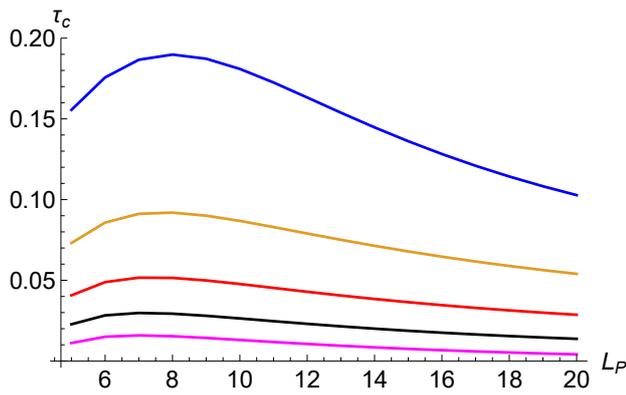


Figure 3: Critical reaction time  $\tau_c$  for the occurrence of a Hopf bifurcation for varying preview length  $L_P$  and control gain  $c_P$ , varying from 0.1rad/m (blue) to 0.5rad/m (magenta) with fixed value  $c_D = 0.02\text{rad s/m}$  for pure steering control.

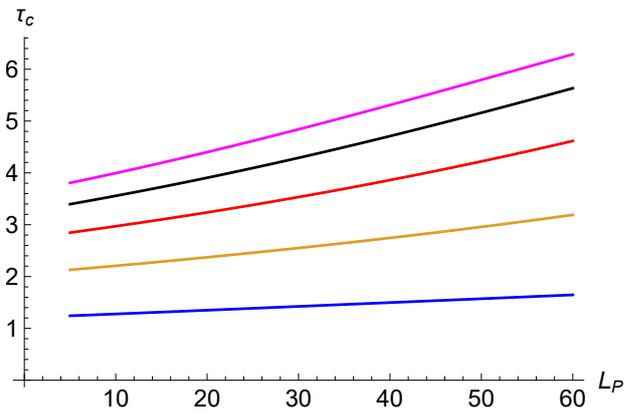


Figure 4: Critical reaction time  $\tau_c$  for the occurrence of a Hopf bifurcation for varying preview length  $L_P$  and control gain  $c_P$  varying from varying from 2N (blue) to 10N (magenta) with fixed value  $c_D = -30\text{Ns}$  for pure Throttle control.

### Further research

We plan to outline realistic estimates for the drivers' control behaviour and explore the resulting driving conditions. Further we will investigate the nonlinear system after loss of stability. In the uncontrolled system considered in [1] we already observed Canard explosions and relaxation oscillations; the feedback control (1) for the drivers' behaviour introduces further time scales into the system dynamics, so we are expecting interesting slow-fast dynamic behaviour of the model.

### References

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- [2] McRuer, D.T.; Graham, B.; Krendel, E.S.; Reisner, W.: *Human Pilot Dynamics in Compensatory Systems*, AFFDL-TR-65-15, 1965.
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- [4] Pacejka, H.B.: *Tire and vehicle dynamics*. Butterworth-Heinemann, Oxford (2012)