Application of the ODE Integration Methods for Multibody Systems With and Without Redundant Constraints

Marcin Pękal* and Marek Wojtyra*
Institute of Aeronautics and Applied Mechanics, Faculty of Power and Aeronautical Engineering, Warsaw University of Technology, Warsaw, Poland

Summary. The multibody approach is commonly used in analyzing complex mechanical systems. Simulation of multibody systems (MBSs) may be a challenging task. Some problems are present for both overconstrained and non-overconstrained MBSs. The other difficulties concern only overconstrained MBSs. Various formulations and numerical solution approaches may be used to simulate MBSs. Some of them allow us to transform the DAEs into ODEs and have different computational properties, especially in the presence of redundant constraints. These methods are investigated in this paper. The numerical methods were compared with respect to the integration method used. The following methods are considered: the Zero Eigenvalue Method, the Pseudo Upper Triangular Decomposition method (two forms—with the Householder Transformation and the Gauss Decomposition), the Schur Decomposition Method, the SVD Method, the QR Decomposition Method, the Direct Integration Method, the Coordinate Partitioning Method, the Udwadia-Kalaba Formulation, the Least-squares Block Solution (two forms), the Udwadia-Phohomsiri Formulation, and the Wang Huston Formulation. For these methods, both the non-stiff ODE solvers (ode45, ode23, ode113) and stiff solvers (ode15s, ode23t, ode23tb, ode23s) of MATLAB® were considered. The methods were tested on two exemplary MBSs described in absolute coordinates—overconstrained and non-overconstrained robotic manipulators.

Introduction and motivation

The method of multibody systems (MBSs) may be used in many areas, e.g., to describe robots, mechanisms, machines, and vehicles. It may also be used for less common applications, e.g., molecular modeling, biomechanical systems, or modeling of contact or impact. The analysis of MBSs may cause some problems.

During simulations of MBSs, various numerical difficulties may occur. One source of the problems may be related to the type of equations to solve. Usually, the MBSs are described by differential-algebraic equations (DAEs). In this set, the algebraic equations describe the constraint equations that represent loop closing kinematic pairs—in the case of relative joint coordinates—or all kinematic pairs—in the case of absolute coordinates.

The DAEs are usually more challenging to solve than ordinary differential equations (ODEs). Depending on the multibody formulation employed, DAEs may have a different index, which determines the computational complexity of the equations. Large DAEs may also be challenging to integrate. Some methods allow us to transform the DAEs into ODEs. These methods are investigated in this paper.

A significant group of MBSs is the group of systems with redundant constraints. The introduction of redundant constraints into the structure of the system is usually the result of a conscious decision of the designer, made after considering the advantages (e.g., greater strength) and disadvantages (e.g., the requirement of greater accuracy of manufacturing of the bodies and the introduction of assembly stresses) of such systems.

Redundant constraints cause further problems in the simulation of the system. In the case of overconstrained MBSs, additional numerical issues result from the fact that the Jacobian/constraint matrix of the considered MBS is rank-deficient. These problems may be reduced by using the appropriate methods to analyze overconstrained MBSs (see, e.g., [2, 1, 5, 6]). Some publications compare selected ODE approaches that are applicable to simulate multibody systems with or without redundant constraints, e.g., [4, 2, 1, 6, 5]. However, a limited number of integration methods were tested in these publications. Hence, a natural question arises: **will the obtained results regarding the relative effectiveness of these methods be the same when other integration schemes are used, and will all integration algorithms give the results at all?** This paper presents the preliminary results of research conducted in this research field.

Numerical methods

In this paper, selected approaches used for the simulation of the MBSs are considered. These methods allow for the integration of the DAEs by using methods for ODEs. The following methods are used: the Zero Eigenvalue Method [4, 6, 2, 1], the Pseudo Upper Triangular Decomposition method (two forms—with the Householder Transformation and the Gauss Decomposition) [4, 6, 2, 1], the Schur Decomposition Method [4, 6, 2, 1], the SVD Method [4, 6, 2, 1], the QR Decomposition Method [4, 6, 2, 1], the Direct Integration Method [5], the Coordinate Partitioning Method [4, 6, 2, 1], the Udwadia-Kalaba Formulation [4, 5, 2, 1], the Least-squares Block Solution (two forms) [5, 2, 1], the Udwadia-Phohomsiri Formulation [5, 2, 1], and the Wang Huston Formulation [6, 2, 1].

Several integration schemes were considered for these methods, i.e., the non-stiff ODE solvers (ode45, ode23, ode113) and stiff solvers (ode15s, ode23t, ode23tb, ode23s) of MATLAB®. The ode45 is the Runge-Kutta method based on the Dormand-Prince (4,5) pair [7], which is frequently the first-choice method [3]. The ode23 is the Runge-Kutta method based on the Bosacki-Shampine (2,3) pair [7]. The ode113 is an application of the Adams-Bashforth-Moulton approach [7]. The ode15s is quasi-constant step size NDFs/BDFs method [7]. The ode23t is trapezoidal rule method [8]. The ode23tb is the Runge-Kutta method with a trapezoidal rule and second-order BDF (TR-BDF2 method) [3]. The ode23s is modified Rosenbrock code [7].

Examples and results

Two MBSs described by absolute coordinates are studied—redundantly constrained and non-redundantly constrained robotic manipulators (see Fig. 1). Both the systems are spatial. The non-overconstrained manipulator was created by removing the redundant body no. 7 of the other MBS. The Euler angles (zxz) are used for the orientation description of the systems to allow us to consider the numerical methods that do not work in the case of a singular mass matrix.
The manipulators are built from the bodies of the same parameters, i.e., length $l = 0.5\ m$, mass $m = 5\ kg$, and moments of inertia $J_x = 0.025\ kgm^2$, $J_y = J_z = 0.12\ kgm^2$. Revolute joints connect the bodies. The gravity acts in the negative direction of axis z_0, and its acceleration $|g| = 9.80665\ m/s^2$. During the simulations, the manipulators were loaded with forces.

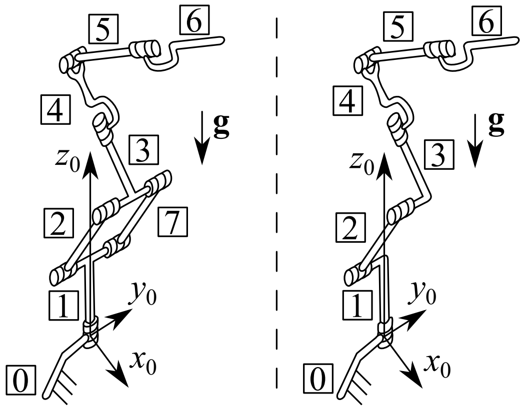

Figure 1: Considered robotic manipulators—overconstrained (on the left) and non-overconstrained (on the right)

The 10-second simulations of motion were performed for selected error tolerance values. MATLAB® run in single-thread mode during the calculations to obtain more reliable time results (which are not disturbed by the time needed to exchange data between multiple threads).

Conclusions

It turns out that the efficiency of the methods depends not only on the selected integration method but also on the structure of MBS. The non-stiff ODE solvers gave results for both the overconstrained and non-overconstrained manipulators. In contrast, the stiff solvers had trouble getting a solution, especially (but not only) in the case of the redundantly constrained manipulator. In addition, not all the methods had produced results in a reasonable time. Some simulations were interrupted when no result was obtained within approximately 15 minutes. It should be emphasized that the obtained results cannot be considered valid for all MBSs. However, based on this work, it can be supposed which of the studied simulation methods will be better for the other systems. This work also provides a reasonable basis for further research in numerical methods for MBSs.

Acknowledgements

This work has been supported by the Warsaw University of Technology under a grant awarded by the Scientific Council for the Discipline of Automatic Control, Electronics and Electrical Engineering.

References

[1] Mariti L., Belfiore N.P., Pennestrì E., Valentini P.P. (2011) Comparison of solution strategies for multibody dynamics equations. *International Journal for Numerical Methods in Engineering* **88(7)**:637-656.

[2] Mariti L., Pennestrì E., Valentini P.P., Belfiore N.P. (2010) Review and Comparison of Solution Strategies for Multibody Dynamics Equations. *The 1st Joint International Conference on Multibody System Dynamics*, Lappeenranta, Finland.

[3] MATLAB® help.

[4] Pennestrì E., Valentini P.P.(2007) Coordinate reduction strategies in multibody dynamics: A review. *Proceedings of the Conference on Multibody System Dynamics*: 1-17.

[5] Pękal M., Frączek J. (2016) Comparison of selected formulations for multibody system dynamics with redundant constraints. *Archive of Mechanical Engineering* **63(1)**:93-112.

[6] Pękal M., Frączek J. (2016) Comparison of natural complement formulations for multibody dynamics. *Journal of Theoretical and Applied Mechanics* **54(4)**:1391-1404.

[7] Shampine L.F., Reichelt M.W. (1997) The MATLAB ODE Suite. *SIAM Journal on Scientific Computing* **18(1)**:1-22.

[8] Shampine L.F., Reichelt M.W., Kierzenka J.A. (1999) Solving Index-1 DAEs in MATLAB and Simulink. *SIAM Review* **41(3)**:538-552.