Harmonic Balance Method for the stationary response of continuous systems with nonlinear hysteretic damping under harmonic excitation

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Summary. Under harmonic excitation, soil exhibits softening behaviour that can be captured through the so-called hyperbolic soil model. The response of systems with such a material model can elegantly be obtained using the classical Harmonic Balance Method (HBM). Soil also exhibits nonlinear hysteretic damping under harmonic excitation, feature which is not incorporated in the hyperbolic soil model. The response of a system that includes also the nonlinear hysteretic damping cannot be obtained using the classical HBM. This work demonstrates the application of an advanced HBM (more specifically, alternating frequency-time HBM) for finite and infinite systems that exhibit softening behaviour and nonlinear hysteretic damping. The purpose of this model is to, in the future, investigate the influence of the nonlinear hysteretic damping on the response of such systems, as opposed to linear viscous or hysteretic damping that is usually adopted. To conclude, we show that the advanced HBM is an effective tool for revealing fundamental characteristics of continuous systems with softening behaviour and nonlinear hysteretic damping whose stationary responses consist of either standing or propagating waves.

Introduction

The Harmonic Balance Method (HBM) is often applied to compute the stationary response of nonlinear *discrete* systems to harmonic loading. It is known to be very efficient as it does not require the simulation of the transient response before reaching the stationary regime, and it can efficiently yield frequency-response curves.

The HBM has been applied to nonlinear *continuous* systems too, but in many cases the nonlinearity is discrete and thus localized at one or multiple points; both finite [1] and infinite [2,3] models have been considered. Furthermore, Chronopoulos [4] presents the general framework for the application of the HBM to an infinite composite structure having distributed but still localized nonlinearity; however, the considered numerical example still only deals with a discrete nonlinear spring connecting two linear parts of the structure. To fill this gap, one of the authors' previous works [5] presents the application of the HBM to continuous systems with distributed nonlinearity, where both finite and semi-infinite systems are considered.

In the previous work [5], the material damping is assumed to be linear. However, soil (among other materials) exhibits nonlinear and hysteretic damping under cyclic loading. For systems with such a material model, the stress-strain relation consists of loading and unloading branches that are different from each other. As the transition points (in the stress-strain relation) from loading to unloading are response dependent and, therefore, not known a priori, the classical HBM, being a purely frequency-based method, cannot be applied. Instead, a so-called alternating frequency-time HBM can be used. Furthermore, the material behaviour described by the so-called Masing model, well-known in soil mechanics, is used to introduce the strain dependence of the shear modulus as well as the nonlinear hysteretic damping, which obviously leads to nonlinearity of non-polynomial type.

To demonstrate the application of this advanced HBM, three canonical problems are investigated (like those in [5]). More specifically, one finite-size system and two semi-infinite systems are considered, and all of them are subject to harmonic excitation at a boundary. The three systems and their stationary responses can be described as follows (see also Figure 1):

a) a 1-D nonlinear layer with a free surface and rigid base: standing shear waves
b) a 1-D nonlinear half-space with a rigid base: vertically propagating shear waves
c) a 2-D axially symmetric nonlinear medium of infinite extent with a circular cavity: radially propagating compressional waves

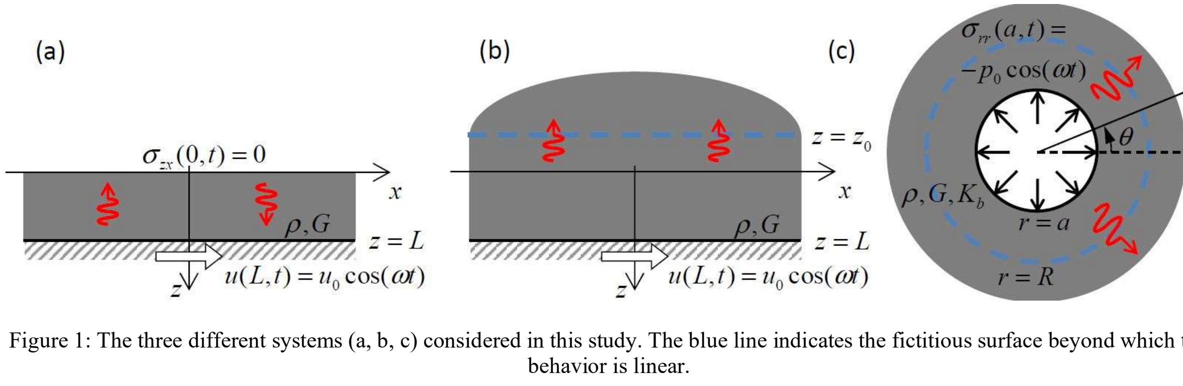

Figure 1: The three different systems (a, b, c) considered in this study. The blue line indicates the fictitious surface beyond which the behavior is linear.

Solution method

The methodology of the alternating frequency-time HBM is as follows. Firstly, an initial guess of the system's response is made based on which the transition points from loading to unloading can be determined. Then, these transition points are assumed to be fixed (not anymore response dependent) and the resulting nonlinear system can then be solved using the classical HBM. This leads to a different response than the initial guess and different transition points from loading to unloading. This process is then repeated until the response has converged. The iterations are performed using a slightly more sophisticated form of the Newton-Raphson method (so-called Levenberg-Marquardt method), which is used to circumvent a potentially singular Jacobian matrix.

As for the spatial discretization, a basic lumping method is used to derive a lattice that mimics the behaviour of the continuous systems (a, b, and c). Additionally, to accommodate the semi-infinite extent of systems b and c, it is possible to identify a fictitious surface beyond which the behaviour is essentially linear (due to the amplitude decay of the waves propagating away from the source). The region beyond that surface is therefore replaced by an exact frequency-dependent non-reflective boundary condition that is applied at the fictitious surface, so that only a finite domain needs to be discretized in the application of the advanced HBM.

Results and conclusion

In Figure 1, we present the response of the system with an expanding cavity (c). The softening behaviour can be observed by the larger amplitude of vibration compared to the linear system. The third harmonic can also be clearly observed, although its amplitude is significantly smaller than the one of the fundamental harmonic; this characteristic has also been observed in [5]. Additionally, it can be seen that the induced nonlinearity is significant from the more than 60% reduction in shear modulus at the cavity surface. Finally, the stress-strain relation follows the Masing model, and it shows that the advanced HBM is capable of handling systems with non-smooth transition in properties.

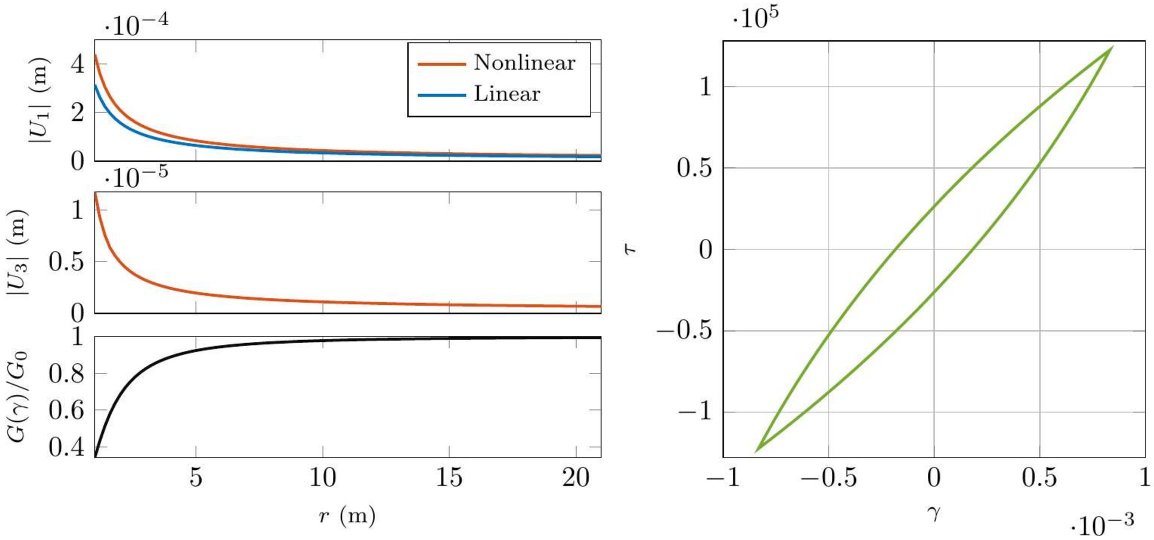

Figure 2: The response of system (c); the amplitude of the first and third harmonics (top and middle panels), the normalized shear modulus (bottom panel), and the stress-strain relation evaluated at the cavity surface (right panel).

To conclude, the advanced HBM is an effective tool for revealing fundamental characteristics of nonlinear continuous systems of finite and semi-infinite dimension that have nonlinear hysteretic damping and whose stationary responses consist of, respectively, standing and propagating waves. The considered systems have applications in earthquake and geotechnical engineering, among others, but the presented methodology is generic.

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