Adaptive Modeling of Coupled Duffing Oscillators Using Machine Learning

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Summary. Tracking parameter variations in mechanical systems is a common and demanding challenge due to various reasons. Conventional approaches leverage measurement of time series signals to remodel the mechanical system to mitigate the impact from parameter variations. However, the effectiveness highly depends on the quality of the measured signals which are affected by the modulation of signals and is prone to noise. Additionally, such methods can exhibit further degraded performance when multiple parameters change simultaneously. With this motivation, we propose a novel adaptive modeling method in this paper. The proposed method aims to explore the possibility of combining both the Science-based Modeling (SBM) and Data-based Modeling (DBM) to develop a hybrid adaptive modeling framework to achieve a higher level of accuracy and broader adaptability. We use two coupled Duffing oscillators as the target model and identify the coupling parameter when it varies. Firstly, the perturbation method is applied to develop a science-based model and obtain the asymptotic solutions of the target system. Based on the asymptotic solutions, the system is parametrized and a series of frequency response plots are obtained corresponding to various coupling parameters. Secondly, the dynamical characteristics of the system like the jump phenomenon can be captured in the series of frequency response plots and extracted as features. Finally, an Artificial Neural Network (ANN) is developed, trained and fine-tuned using the features to identify the changing coupling parameter. The results compared with a baseline grey box model demonstrate the effectiveness of the proposed method in increasing the accuracy and robustness of adaptive modeling.

Background Introduction

A model is a abstracted description of a real - often complex - dynamic system. The accuracy and adaptability of a model is critical for understanding of observed phenomena, predictions of system behaviors as well as design, optimization, diagnostics and control. We call the classical approach of modeling, Science Based Modeling (SBM), that used the traditional mathematical models and the associated computer codes have certainly developed reasonable verisimilitude with real behaviors. Generally, the SBM method uses physics or established rules, which are capable of capturing some observed effects qualitatively and provide useful insights into the underlying causalities. However, the limitation of SBM is derived from the fact that many of the practical physical systems are becoming increasingly more complex than can be modeled accurately with the physics we know. On the other hand, due to accelerating development of machine learning, sensor technology, computer hardware and big data, the uncertainties and variations of the systems are more reflected in the collected data. Therefore, these inspire a novel approach of modeling physical systems, namely, Data Based Modeling (DBM). Typically, the DBM method establishes a pattern from the observed data to predict the system response for future inputs. DBM highly depends on the quality and quantity of collected data and the nature of the system, so obtaining more effective data is key to enhance the performance of DBM method. However, the inherent limitation of DBM comes from the fact that it cannot be easily parameterized, or extended to situations that the model has not been exposed to. Any tiny changes of the system can compromise the validity of data-based modela such as system degradation, environmental changes and operational condition alterations, all of which are inevitable in practice. Therefore, it is of significant importance to develop adaptive modeling techniques that can adapt to changes and are applicable to real complex systems [1].

Target system

In this paper, we develop an adaptive modeling method and demonstrate it by applying to a set of two coupled duffing oscillators[2, 3], the governing equations of the system are shown in Eq. (1).

$$\begin{aligned} x'' + k_1^* x + d_1^* x' + k_2^* x^3 + k_3^*(x-y) = 0 \\ y'' + k_1^* x + d_1^* y' + k_2^* x^3 + k_3^*(y-x) = 0 \end{aligned} \tag{1}$$

We select such a system because it is a low-order baseline system which is nevertheless complex from the point of view of nonlinearity. Moreover, such a model is the candidate for description of various mechanical and electro-mechanical systems. The coupling coefficient of real connected systems is one of the most variable and least predictable coefficients particularly in higher order systems. Therefore, in our target system, it is set to be an unknown parameter and requires identification under varying conditions to evaluate the effectiveness and generality of the proposed method. The analysis is also conducted with different parameters set of damping and nonlinearity to verify the generality of the method. As the external force can vary due to uncontrollable loads of the system, we randomly change the amplitude of excitation within a fixed range.

Methodology

The overview of the hybrid adaptive modeling framework is shown in Fig. 1. Assuming the physical system is perfectly modeled, the governing equations are obtained at time t_1. At time t_2, the coupling coefficient of the system may change

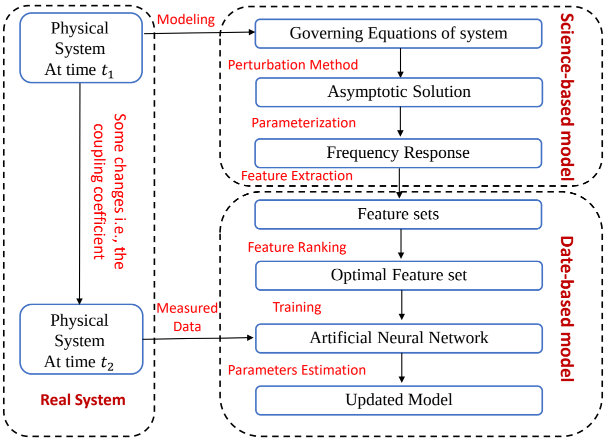

Figure 1: Overview of proposed approach for predicting coupling coefficient

due to wear, fatigue or other operating conditions. Note that t_1 and t_2 are orders of magnitude larger than system dynamic response times. To retain the accuracy of the model, it is necessary to track the variations of the coupling coefficient to update the previous model. We use a perturbation method to analytically solve the governing equations and obtain the frequency response by changing the frequency of excitation. Subsequently, the frequency response corresponding to different coupling coefficients are considered. The nonlinear dynamical characteristics like jump phenomenon and bifurcation points are extracted as appropriate features to describe the changes of frequency responses with the variations of the coupling coefficient. In order to prevent the interference of other parameters like nonlinear stiffness and damping, the mutual information [4] between features and other parameters are also determined. The features which are more sensitive to the changes of the coupling coefficient and less sensitive to other parameters are chosen as an appropriate feature set. Subsequently, an artificial neural network (ANN) with one hidden layer and twenty nodes is developed and trained by selected features in which the architecture of the ANN is determined by grid search, and the performance is optimized by regularization and cross validation. Finally, the frequency response of the real physical system at time t_2 is measured which is more noise-resistant than time series signals. The trained ANN takes the features of measured frequency response signal as input to predict the changed coupling coefficient with 99.8% accuracy.

Conclusions

This paper proposes a hybrid adaptive modeling to identify the changing coupling coefficient in two coupled Duffing oscillators. The features extracted from the Science Based Model are more efficient to capture the nonlinear dynamical characteristics of the real system and help improve the prediction from the Data Based Model. The application of mutual information and feature ranking improves the robustness of extracted features. The ANN developed and trained by the optimal feature set demonstrates better ability to identify the changed coupling coefficient with much higher regression than the classical grey box model.

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