Development and Parameter Estimation of a Low Order Model of a Hyperelastic Plate Exhibiting 2:1 Resonant Response

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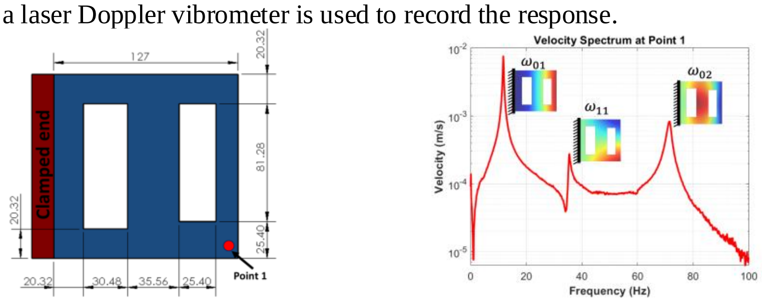
Summary. This work presents results of experimental investigation and a low-order model development for resonant complex dynamics of a cantilever macro-plate. The plate is fabricated from a 'hyperelastic' material using 3D printing technology. The geometry of the plate with cut-outs is optimized such that the second linear bending mode frequency ω_{02} is nearly twice the linear first twisting mode frequency ω_{11}. Based on the observed 2:1 resonant response under harmonic excitation near ω_{02}, a low-order 2 DOF dynamic model is developed to simulate the plate response. The unknown model parameters are extracted using Harmonic Balance solutions and curve fitting techniques. A good agreement is observed between the analytical and the experimental results for different excitation levels.

Introduction

Modal coupling and their influence on system response continues to attract much interest and has been the subject of several review papers that study the influence of mode interactions on different structures from the macro to the micro scale and their potential applications [1]. Internal resonance is among the primary nonlinear coupling mechanisms between the different modes of vibration which can be triggered when the ratio between the frequencies of coupled modes is commensurate and the directly excited response amplitude exceeds a certain threshold [1]. Parameter estimation techniques have received significant attention for developing models that can predict the complex dynamics of a structure under loading conditions that are difficult to test experimentally [2]. Most of the prediction techniques rely on error minimizing algorithms that minimize the difference between predictions of the analytical model and the experimental results. Conventionally, one system state is experimentally measured, and the other states are derived using numerical integration or differentiation techniques which amplify noise at low or high frequency. Processing the signal to reduce the high frequency noise might remove the response at higher harmonics and also introduce aliasing in the data [2]. Frequency response approach based on curve fitting eliminates the issues associated with signal processing but requires more theoretical effort and a comprehensive understanding of the system model and the effects of different coefficients on the response [2]. In this work, the plate geometry, Fig. 1a, is designed following the topology optimization procedure described in [3] such that for the cantilever plate, the second bending mode ω_{02} $(71\ Hz)$ and the first twisting mode ω_{11} $(35.5\ Hz)$ are in 2:1 ratio. The plate is fabricated using 3D printing technology from a hyperelastic material thus incorporating material nonlinearity. To study the plate dynamics, a TIRA shaker is used to actuate the plate at different acceleration levels and a laser Doppler vibrometer is used to record the response.



(a) (b)

Figure 1: (a) The dimensions of the optimized plate are in millimeters showing the laser measurements point. Plate thickness = $1.27\ mm$. (b) The FFT of the impulse response showing the first three modes of vibration (insets: Corresponding mode shapes).

Results and discussion

To capture the modal interaction of the two modes in the cantilever plate response, two coupled oscillators described by generalized coordinates $u(t)$ and $v(t)$ are considered. The directly driven mode $u(t)$ is modeled as nonlinear oscillator with linear natural frequency ω_{02} along with cubic $\alpha_c u^3(t)$ and quadratic $\alpha_q u^2(t)$ nonlinear stiffness forces. The experimental data for the directly excited mode shows a nonlinear dependence of the damping on the amplitude of vibration implying that a nonlinear dissipation mechanism needs to be incorporated. Here, a nonlinear damping model compromising of quadratic displacement multiplied by the velocity $u^2(t)\dot{u}(t)$ is assumed. The secondary mode amplitude $v(t)$, the mode only excited due to its coupling with the primary mode, is modeled with a linear oscillator of frequency ω_{11} which equals to half the driven mode frequency ω_{02}, and a nonlinear interaction term. The interaction between the two oscillators is assumed to be nonlinear in the form of $\big(v(t)\big)^2$ acting on the driven oscillator and $u(t)v(t)$ acting on the secondary oscillator. The equations of motion normalized by the modal masses $m_{1,2}$ are then:

$$\ddot{u}(t) + 2\xi_2\omega_{02}\dot{u}(t) + (\omega_{02})^2u(t) + \alpha_q u^2(t) + \alpha_c u^3(t) + 2\xi_{non}\omega_{02}u^2(t)\dot{u}(t) = \gamma_2\big(v(t)\big)^2 + rA\cos(\Omega t) \tag{1}$$

$$\ddot{v}(t) + 2\zeta_1\omega_{11}\dot{v}(t) + (\omega_{11})^2v(t) = \gamma_1 u(t)v(t) \tag{2}$$

where ξ_2, ξ_1 are the modal damping ratios acting on the driven and the secondary oscillators, respectively. ξ_{non} is the nonlinear damping coefficient. γ_2, γ_1 are the coupling coefficients between the two modal amplitudes, A is the amplitude of the base excitation acceleration, and r is the projected modal force acting on the driven oscillator. To extract the unknown parameters, the system of equations are initially numerically integrated for different values of the parameters and the numerical results are compared with experimentally recorded data. Also, the effect of each parameter on the final response is studied which helps in understanding its influence on the system response. Subsequently, the harmonic balance method is used to solve for steady state solutions of Eq. (1) and Eq. (2) in which the response is assumed to have the form of a truncated Fourier series. In the current analysis, two harmonics plus the constant term are utilized. To find the unknown parameters, the parameter extraction procedure is divided into multiple steps. The resonant frequencies values ω_{02} and ω_{11} can be inferred from the impulse response results given in Fig. 1b and are found to be $\omega_{11} = 35.5\ Hz$ and $\omega_{02} = 71\ Hz$. Then, the response in which only the directly excited mode has non-zero response and the internal resonance has not yet activated is considered, see Fig. 2 for the case of 1.65g base excitation. The system of equations assuming zero coupling coefficients, that is, γ_1 and γ_2 are set equal to zero, are solved for different values of the unknown parameters ξ_2, α_q, α_c, ξ_{non}, and r until the simulation results match the experimentally recorded frequency response, as shown in Fig. 2. To extract the coupling coefficients γ_1 and γ_2, the experimental response at 1.75 g, Fig. 3, is considered, and the 2 DOF model in Eq. (1) and Eq. (2) is solved for different sets of coupling coefficients until error is minimized between the harmonic balance results and the experimental response curve. Summary of the all the extracted parameters is given in Table 1. To verify the 2DOF model with the extracted parameter values, Eq. (1) and Eq. (2) are used for higher excitation levels and the simulation results are compared with the corresponding experimental data. As shown in Fig. 4, the harmonic balance results at various excitation levels are in reasonable agreements with the experimental measurements.

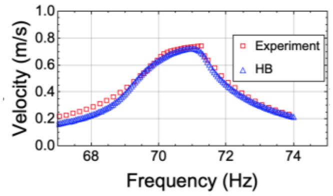
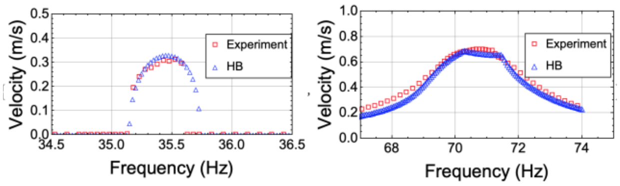
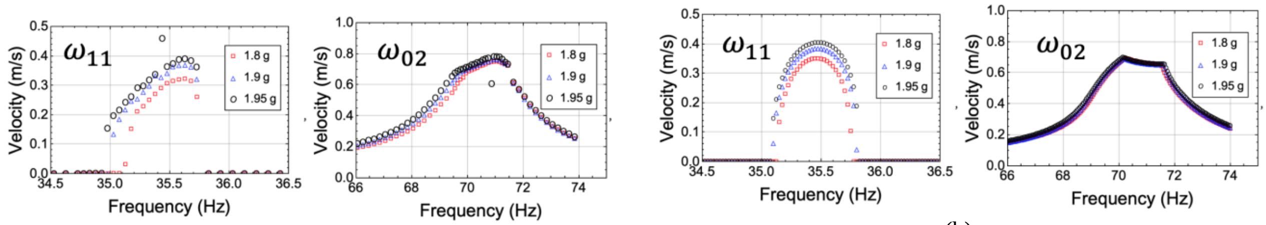


Figure 2: Experimental and harmonic balance frequency response results (HB) for the 1.65 g excitation case.



(a) (b)

Figure 3: Experimental and harmonic balance frequency response results (HB) for the 1.75 g excitation case with internal resonance activated. (a) Near ω_{02}. (b) Near ω_{11}.



(a) (b)

Fig. 1: Frequency response of the macroplate to a harmonic base excitation at different acceleration levels with internal resonance activated. (a) Experimentally recorded results. (b) Harmonic balance results.

Table 1: Summary of the extracted parameters.

Parameter	ω_{11}	ω_{02}	ξ_2	α_q	α_c	ξ_{non}	r	γ_1	γ_2	ξ_1
Value	35.5	71	2.5×10^{-3}	6×10^{7}	1.54×10^{10}	1.5×10^{4}	0.525	1.45×10^{7}	2.3×10^{6}	0.1

Summary and Conclusions

A two-mode nonlinear model to predict the resonant response of a 3D printed cantilever macroplate to a base excitation is developed. The system is modeled with two nonlinearly coupled oscillators. The plate is designed such that the linear natural frequencies ω_{02} and $\omega_{11}$are in 2:1 frequency ratio. A harmonic balance analysis is used to approximate the response of the coupled oscillators using curve fitting techniques to extract the unknown parameters. The parameter estimation procedure is divided into three steps. In each step, the experimentally recorded results are used to find a subset of the unknown parameters. The final estimated parameters are used to predict the response at higher excitation levels. The analytical model predictions are seen in good agreement with experimental data.

Acknowledgments

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