Resonant nonlinear triad interactions of acoustic–gravity waves

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Summary. Acoustic waves, such as underwater sounds generated by earth-plate movements, and gravity waves, such as surface ocean waves, are two types of waves that are thought to share very little in common. However, recently it has been shown theoretically that acoustic–gravity waves can interact and share energy. Such interaction could explain natural phenomena such as microseisms (faint earth tremor), but also has many implications from tsunami mitigation and energy harnessing, to creating new measurement techniques that can be applied in invasive medical operations. In this talk I will present a review on nonlinear interaction of acoustic–gravity waves, theory and applications.

Background

Acoustic (compression) waves and free-surface (gravity) waves are virtually decoupled for two main reasons. Firstly, the speed of sound in water far exceeds the maximum phase speed of gravity waves. Secondly, the mode shape with depth is oscillatory for acoustic modes, and exponentially decaying for gravity waves. Nevertheless, it has been argued theoretically that these two types of wave motion could exchange energy via resonant triad nonlinear interactions [1, 2, 3, 4, 5]. There are two cases of interest this review talk focuses on: (I) two gravity waves interacting with an acoustic mode of a comparable frequency (almost double) [1, 2, 3]; and (II) two acoustic modes interact with a gravity wave of a comparable lengthscale [4, 5]. In the first case, the theory suggests that for a perfectly tuned triad almost all energy initially stored in the gravity waves can transfer into the generated acoustic mode, whereas for wavepackets a maximum of 40% energy transfer can be obtained [2]. This has implications at the ocean scale where interacting surface gravity waves can generate microseism (faint earth tremor) deep at the ocean floor [6, 7]. Not less interestingly is the particular solution where two gravity waves of identical frequency generate a standing acoustic mode [8]. Such setting might explain a physical phenomenon known as time reversal [9, 10]. The same solution might explain the evolution of Faraday waves [8, 11] that find various applications in physics. In the second case, the interaction of two acoustic modes with one gravity wave has implications on underwater communication [4], wave energy harnessing, or more ambitiously tsunami mitigation [5].

Amplitude evolution equations

We consider the propagation of surface-gravity waves interacting with acoustic wave disturbances in water of constant depth over a rigid bottom. The equation governing the velocity potential in the fluid interior is obtained by combining continuity with the unsteady Bernoulli equation, i.e cubic nonlinear wave equation. The boundary conditions are the standard higher order kinematic and dynamic conditions, at the surface; and the no-penetration condition at the bottom. In the first case, resonance is possible among two surface -gravity waves and a single acoustic mode. The conditions for resonance comprise an interplay of the frequencies $\sigma_+ + \sigma_- = \omega$, and wavenumbers $k_+ + k_- = \kappa$, where $\sigma_\pm$ and $k_\pm$ represent the two gravity waves, which combined form the acoustic mode represented by ω and κ. To derive the evolution equations we employ multiple-scale analysis, which yields

$$\frac{\partial A}{\partial t} + c_1 \nabla A + c_2 \nabla^2 A \propto S_+ S_-, \qquad \frac{\partial S_\pm}{\partial \tau} \propto A S_\mp^* + [\text{cubic terms}] \tag{1}$$

where A and $S_\pm$ are the amplitudes of the acoustic and two gravity waves, t is the interaction timescale, ∇ is the gradient (∂_x, ∂_y), and c_1 and c_2 are constants. The derived evolution equations allow quantifying the parameters (i.e. frequency, wavelength, and amplitude) needed to finely tune the interaction, which controls the energy exchange. Following a similar approach we derive the amplitude evolution equations for two acoustic modes interacting with a gravity wave. Now, the conditions for resonance become $\omega_+ + \omega_- = \sigma$, and wavenumbers $\kappa_+ + \kappa_- = k$ and the evolution equations are fundamentally different. The following are some examples that will be discussed.

Example 1: Faraday Waves

This case is analogous to a surface gravity disturbance (Gaussian) of frequency ω over a fluid layer that is subject to a continuous vertical oscillation, e.g. due to underwater tremor, at double the frequency. The interaction excites subharmonic standing field of Faraday-type waves of frequency 2ω, as shown in the figure 1 (from [11]).

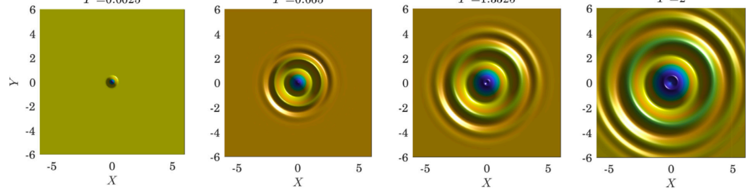

Figure 1: Evolution of Faraday-type waves from a gravity disturbance interacting with a long-crested acoustic mode [11].

Example 2: Time-Reversal

A mathematical model for the evolution of a time-reversed gravity wave packet from a nonlinear resonant triad perspective is derived [8]. Here the sudden appearance of an acoustic mode is analogous to a sudden vertical oscillation of the liquid film, which resonates with the original surface-gravity wave packet causing the generation of an oppositely propagating (time-reversed) surface-gravity wave of an almost identical shape, see figure 2

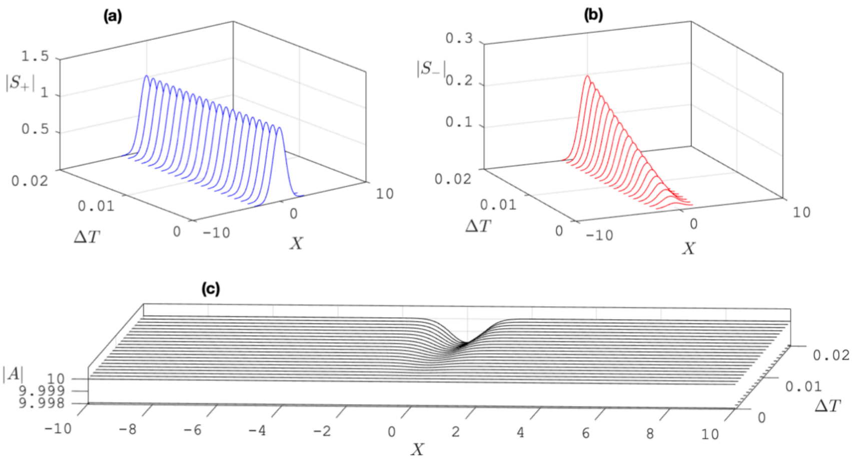

Figure 2: Amplitude evolution of time reversal triad: (a) original disturbance; (b) time reversed disturbance; (c) sudden acoustic mode.

Example 3: Tsunami Mitigation

A tsunami interaction nonlinearly with two acoustic modes. The tsunami envelope is redistributed behind over a larger space and its amplitude is reduced, see figure 3.

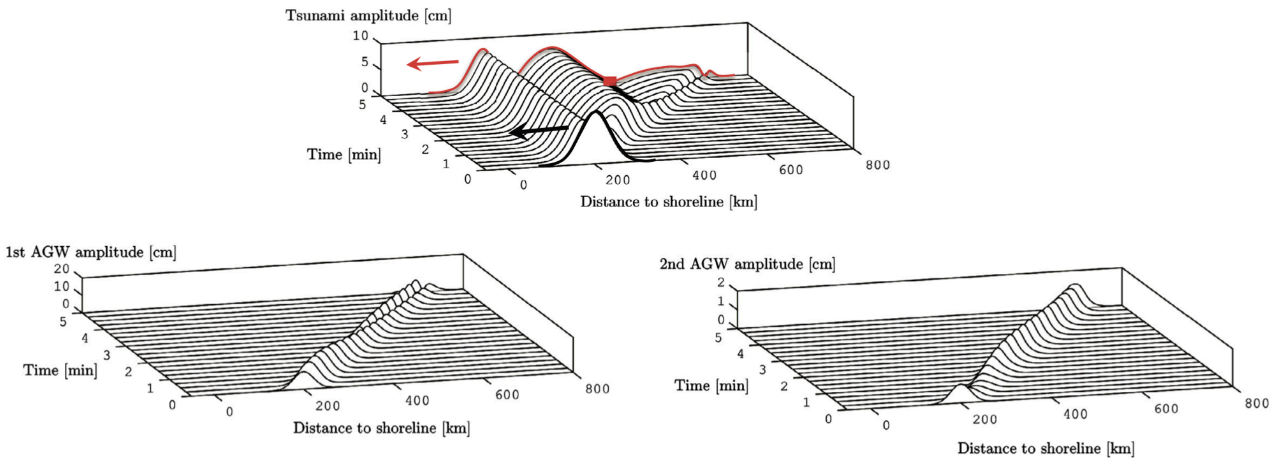

Figure 3: Amplitude evolution. A tsunami propagates from right to left (top), exchanges energy with two acoustic waves (middle and bottom), that propagate from left to right, [5].

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