Bifurcations and stability transitions in nonlinear dynamics of a planar undulating magnetic microswimmer

Jithu Paul*, Yizhar Or* and Oleg Gendelman*

**Faculty of Mechanical Engineering, Technion–Israel Institute of Technology, Haifa, Israel*

Summary. *A microswimmer, motivated from biological microorganisms or cells motion, which has two links representing a tail and a magnetized head is studied. The links are connected by a passive elastic joint and the microswimmer is actuated by an external time – periodic magnetic field. This simple system is very rich in dynamics and we identified that there exists co-existing periodic solutions-symmetric as well as asymmetric, and stability transitions with pitchfork bifurcations. There is optimum observations for the displacement and velocity of the swimmer with respect to the system parameters, may enable significantly improving the design aspects of the robotic microswimmer.*

The micro swimmers are motivated by biology [1], and their dynamics is always interesting . Among the simplest and most efficient techniques for actuation of robotic microswimmers , is by applying time-varying external magnetic fields [2]. One of the important milestones in the field of magnetically actuated microswimmer is given in by Dreyfus et. al [3]. Inspired from the classical work of Purcell [4], a very simple model of two-link swimmer has been presented by Gutman et. al [5], with an external magnetic actuation on the head as shown in Fig. 1. The external magnetic field, $B(t) = [\gamma, \beta\sin(\omega t)]^T$ has an oscillating component in y direction and a constant x component, γ. The same model with paramagnetic excitation is studied in [6] with interesting observations on bistability and analogy of tilted Kapitza pendulum. We revisit the model in [5] and explore co-existence of periodic solutions as well as their bifurcations, stability transitions, and symmetry-breaking.

The system in Fig. 1 has four coordinates: body position (x,y) and two angles θ and ϕ, which represent body orientation angle and relative angle between the links, respectively. Invariance with respect to x and y enables reduction of the system into two dimensional state equations in two variables, θ and ϕ only. The stiffness of the torsional spring at the passive elastic joint is given by k. Coming to the major assumptions, the swimmer is submerged in a Newtonian fluid and it remains neutrally buoyant (no effect due to gravity). The micron size of the swimmer allows assuming low Reynolds number hydrodynamics where viscous forces dominate while inertial effects are negligible. The resistive force theory for slender bodies governs the net forces and torques acting at the links' centers due to viscous drag. In the work of Gutman et. al [5], fixed, $\gamma = 1$ and small $\beta \ll \gamma$ are assumed, and focused only on symmetric periodic solutions with mean $(\theta, \phi) = (0,0)$.

Here, as an extension to [5], as shown in Fig. 2, we extracted other co-existing solutions, symmetric around mean $(\theta, \phi) = (\pi, 0)$, as well as asymmetric, and their stability transitions (Fig. 2(a)) and bifurcation (Fig. 2(b)). Upon varying the actuation parameters $(\beta,\ \gamma, \omega)$, the results are obtained by numerical analysis using the MATLAB tools ODE45 and fsolve. There are also optimum plots (for stable region) for X, net displacement in x versus β , amplitudes of excitation (Fig. 2(c)) and V, mean velocity versus ω, frequency (Fig. 2(d)), where all the parameters are in experimentally relevant region. It can be analytically studied since there exists small amplitudes for $\theta,\ \phi$ for $\gamma \ll 1$. These stable optimum regions and indications towards stability transitions can improve designing a simple efficient two link robotic microswimmer, which can be used for a wide range of applications.

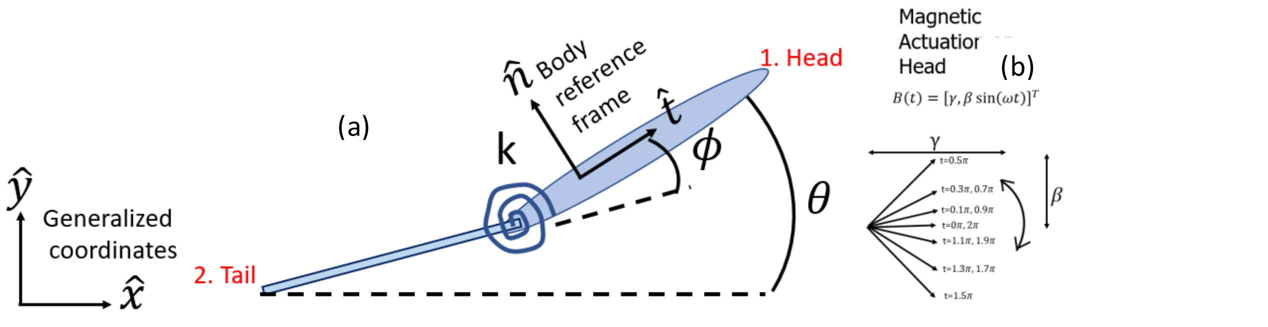

Fig. 1. Model system, (a) shows two link microswimmer which are connected by torsional spring and its head receives magnetic actuation. (b) shows the details of magnetic actuation

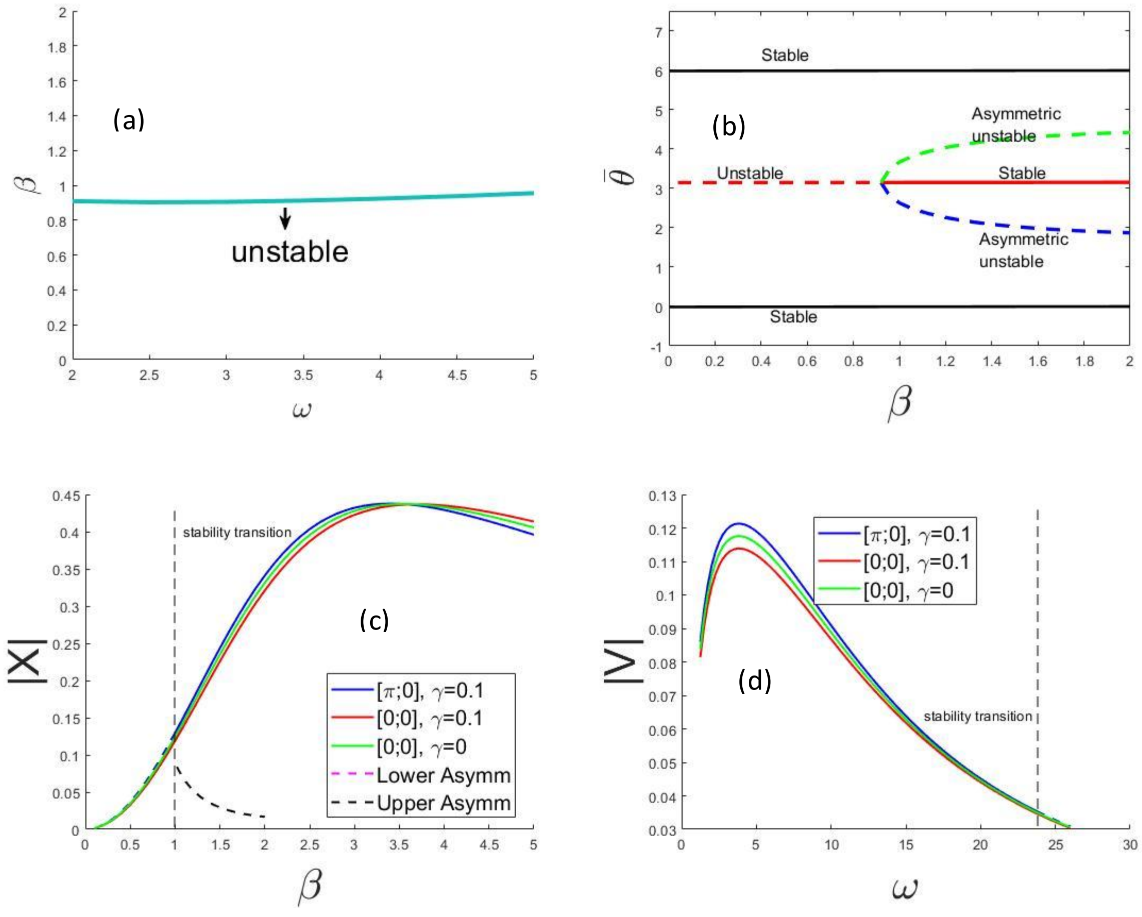

Fig. 2. (a) shows Stability transition in $\beta - \omega$ plane at $\gamma = 0.1$, (b) shows possibility of a subcritical pitchfork bifurcation at the stability transition where $\omega = 2, \gamma = 0.1$, (c) shows optimum displacement plot with respect to amplitude of actuation oscillations β and (d) shows the optimum velocity plot versus frequency ω.

References

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