Period Tripling States and Non-Monotonic Energy Dissipation in MEMS with Internal Resonance

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Summary. We study free ringdown dynamics of a MEMS system with two modes near a 1:3 internal resonance. By separately preparing initial states and measuring the motion of both modes, we demonstrate that dependent on the initial relative phase the modes can either bypass or enter a phase-locked state, which can persist several times longer than the dissipation timescales. The sustained energy transfer between modes in the phase-locked state leads to non-monotonic energy dependence for one of the modes and overall lower dissipation rate for the system. The observations are accurately modeled by the coupled equations of motion, and can be understood by considering the low frequency mode as entering or bypassing a period tripling state under the influence of the periodic force from the high-frequency mode.

Persistent phase-locked state is described by a PTS model.

Nonlinearity in MEMS resonators is widely used in frequency stabilization [1], dissipation engineering [2], and improvement of resonance-based sensing [3] to achieve performance not available in the linear regime. Here, we thoroughly studied the nonlinear dynamics of two coupled MEMS resonators during free ringdown. The two resonators are designed with commensurate 1:3 eigenfrequencies to facilitate fast energy flow at internal resonance (IR). Remarkably, the two modes can get locked during ringdown and persist in this phase-locked state for extended periods of time, much longer than their intrinsic dissipation timescales. During relaxation, the low-frequency mode exhibits striking non-monotonic energy dissipation and negative modal energy dissipation rate (transient energy gain). The rich dynamics observed in the experiment are well explained by an intuitive model that regards the low-frequency mode at period-tripling states (PTS) created by the high-frequency mode, similar to the period-two states in parametric oscillators. In contrast to works at steady states [1,4,5], we demonstrate for the first time the phase lock at the transient states, the non-monotonic dissipation rate, and we provide an intuitive PTS model to describe it, simultaneously and independently from [5]. The observation and the model pave the way to engineering efficient energy flow between coupled nonlinear MEMS, which can be used for frequency stabilization and dissipation engineering.

Results

The two coupled modes are the fundamental in-plane mode (mode1) and torsional mode (mode2) of a clamp-clamp beam, as shown in Figure 1. The two modes are driven electrically with driving frequency $\omega_{1,osc}$ and $\omega_{2,osc}$ via the side gates. The modes are measured electrically and optically by an oscilloscope and a vibrometer, respectively. The eigenfrequencies of mode1 and mode2 are $\omega_1/2\pi \approx 64.6$ kHz and $\omega_2/2\pi \approx 199.9$ kHz ($\omega_1 \approx \omega_2/3$). When the driving force is strong, they exhibit spring hardening and softening effects, respectively, as shown in Figure 2. The dip on mode1's spectrum (orange) corresponds to the IR frequency of $\omega_{1,osc} = \omega_2/3$ where the model coupling is the strongest. In the experiment, we drive the two modes separately to their initial amplitude $A_{1,0}$ and $A_{2,0}$, labeled as red dots. At time $t = 0$, we turn off their drive simultaneously to let them ring down. Following their Duffing trajectory (black arrows in Figure 2), their frequencies shift due to the exponentially decaying amplitude (Fig. 3) and finally get locked at IR (black dots in Figure 2). The locking duration is $\approx 3\times$ of the system intrinsic dissipation time $\approx 1/\Gamma_2$ where $\Gamma_2/2\pi \approx 3.3$ Hz is the intrinsic dissipation rate of mode2. The two modes persist in this state by spontaneously transferring energy from mode2 to mode1, indicated by the energy gain and rapid energy loss of mode1 and mode2 in Fig. 3(b). Their energies are calibrated to the same scale and fitted (black line) based on energy conservation in this system. The coupled two modes system can be regarded as a parametric resonator at PTS (mode1) subject to a period-three harmonic drive (mode 2). Repeating the same ringdown-locking experiment, we find the system exhibits discrete relative phase $n\times2\pi/3$ shown in Figure 4, similar to period-two parametric oscillators. Following this logic, the non-monotonic and negative energy dissipation rate of mode1 (Γ_1) shown in the inset of Figure 3(b)) can be explained by the opposite Duffing coefficients of the two modes: (1) Spring softening effect of mode2 leads to increasing ω_2 during ringdown. (2) Locked by mode2, mode1's amplitude increases at higher frequency following its spring hardening spectrum.

Discussion

More generally, the gain or loss of mode1 can be engineered by altering mode2's Duffing coefficient, i.e. mode1 can either perform gain or rapid loss depending on mode2's parameters. The findings of phase-locking states and tunable dissipation rates are useful for energy dissipation engineering, such as for fast switches or low-dissipation timing/sensing devices. The proposed intuitive model provides a picture to understand the complicated dynamics in coupled nonlinear resonators. More details can be found in our preprint [6].

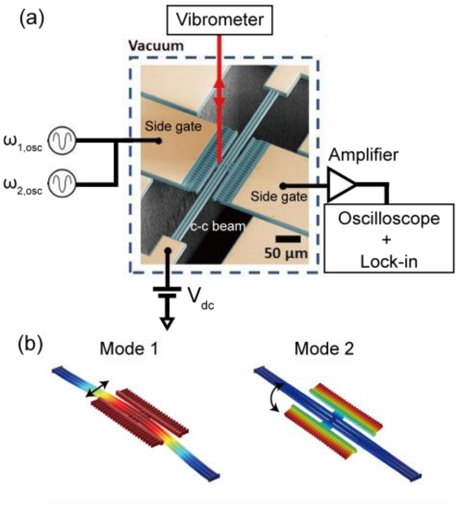

Figure 1: (a) Measurement schematics and false-colored SEM micrograph of the clamp-clamp beam MEMS. Optical and electrical measurements are performed, simultaneously. (b) Simulated mode shape of the two coupled modes.

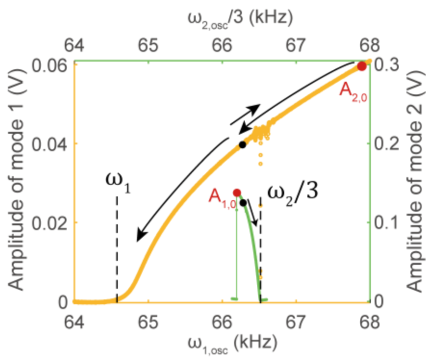

Figure 2. Spectrums of mode1 and mode2 are labeled by yellow and green dots. The oscillating frequency of mode2 is divided by 3. The two modes have opposite Duffing coefficients. In ringdown experiments, we set the initial conditions at $A_{1,0}$ and $A_{2,0}$ (red dots), respectively, and turn off the drive simultaneously. They evolve to equilibrium following the black arrows. After locked at the black dots, mode1 experiences frequency and amplitude increase shown as the short black arrow.

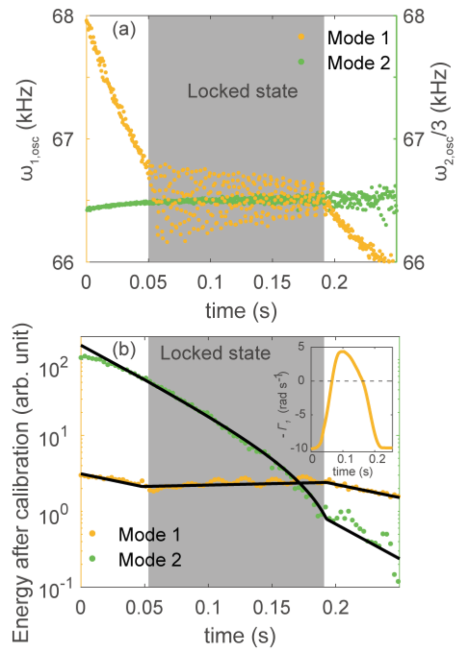

Figure 3. Oscillating frequencies and energy of the two modes are presented in (a) and (b), respectively. The yellow and green dots correspond to mode1 and mode2, respectively. The inset of (b) shows the minors of the measured effective dissipation rate of mode1 ($-\Gamma_1$). It indicates that mode1 shows a non-monotonic and negative dissipation rate during locking.

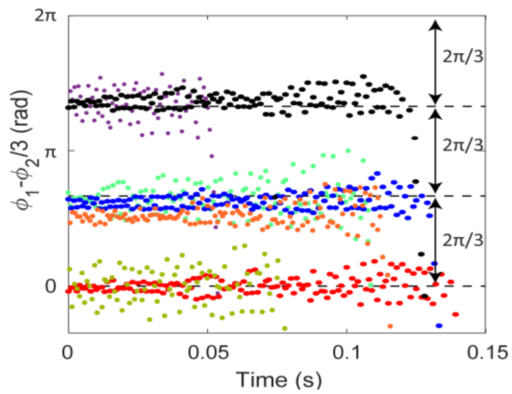

Figure 4. Relative phase of mode1 (φ_1) and mode2 (φ_2) in 7 repeating experiments. φ_1-φ_2/3 exhibits discrete values of (0, 2π/3, 4π/3) as expected in period tripling states.

References

[1] Antonio D., Zanette D. H, López D. (2012) Frequency stabilization in nonlinear micromechanical oscillators. *Nature* communications **3**(1), 1-6.

[2] Güttinger J., et al, (2017) Energy-dependent path of dissipation in nanomechanical resonators. *Nature nanotechnology* **12**(7), 631-636.

[3] Huang, L., et al, (2019) Frequency stabilization and noise-induced spectral narrowing in resonators with zero dispersion. *Nature* communications **10**(1), 1-10

[4] Czaplewski D. A., et al, (2018) Frequency comb generation in a nonlinear resonator through mode coupling using a single tone driving signal. *Technical digest-Solid-State Sensor, Actuator, and Microsystems Workshop*

[5] Yan Y., et.al. (2022) Energy Transfer into Period-Tripled States in Coupled Electromechanical Modes at Internal Resonance. *PRX* in press, arXiv:2206.0163

[6] Wang M., et al, (2022) Persistent nonlinear phase-locking and non-monotonic energy dissipation in micromechanical resonators. arXiv: 2206.01089