ANALYSIS OF COUPLED NON-LINEAR OSCILLATORS BY THE ASYMPTOTIC NUMERICAL METHOD

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Summary. This study is based on the theoretical and experimental works of Cadiou et al. [1, 2] who designed a non-linear vibration absorber using an electro-magnetomechanical coupling. The present work limits itself to the pure mechanical response of a 2 degrees-of-freedom oscillator with a cubic non-linear coupling between the Linear Oscillator (LO) and the non-linear energy sink (NES). The numerical analysis is performed using the Harmonic Balance Method (HBM) in association with the Asymptotic Numerical Method (ANM), following the initial idea proposed by Cochelin et al. [3]. This approach offers an understanding of resonance mechanisms of the LO and energy transfers to the NES. Numerical experiments are performed in order to identify influence of the parameter values. Numerical results are also compared to theoretical ones. They confirm the relevancy of the numerical approach which may be retained to the design of a non-linear vibration absorber.

Extended Abstract

Non-linear energy sinks (NES) are known for their efficiency in the vibration mitigation as they do not have to be tuned to the natural frequency of the supporting structure. In this study, a 2 degrees-of-freedon (DOF) model is analysed (Fig.1). It is composed of a Linear Oscillator (LO) non-linearly coupled with the absorber (NES). The LO is defined with its mass m_1, a constant stiffness k_1 and a viscous damping parameter c_1. The absorber is defined through its mass m_2, its viscous damping parameter c_2 and its cubic stiffness k_{2c}. The LO is directly excited by an harmonic force $f(t) = Fcos(\omega t)$. Finally, $x_1(t)$ and $x_2(t)$ denote the LO and NES displacement, respectively. Thus, the problem to be solved reads:

$$\left\{ \begin{array}{c} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_{2c}(x_1 - x_2)^3 = Fcos(\omega t) \\ m_2\ddot{x}_2 - c_2(\dot{x}_1 - \dot{x}_2) - k_{2c}(x_1 - x_2)^3 = 0 \end{array} \right.$$

In this study, the problem is solved coupling the Asymptotic Numerical Method (ANM) with the Harmonic Balance Method (HBM). The former is a continuation technique based on Taylor series of the unknowns while the later is a decomposition of unknowns into truncated Fourier series.
In order to efficiently apply the ANM, quadratic recast of equations is a preliminary stage. So, auxiliary variables $y_k = x_k^2$ and velocity variables $v_k = \dot{x}_k$ $(k = 1, 2)$ are defined and the non-linear dynamic problem formally reads:

$$\underline{\dot{Y}} = \underline{G}(\underline{Y}, \omega) \text{ with: } \underline{Y} = \{x_1, x_2, v_1, v_2, y_1, y_2\}^\top$$

The HBM is then applied and the unknown $\underline{Y}$ is decomposed into truncated Fourier series with H harmonics:

$$\underline{Y} = \underline{Y}_0 + \sum_{p=1}^{H} \left[\underline{Y}_p^c \cos(p\omega t) + \underline{Y}_p^s \sin(p\omega t)\right]$$

Fourier series are introduced into the non-linear dynamic problem. After balancing terms of the same harmonic index, the components $\underline{Y}_0, \underline{Y}_p^c$ and $\underline{Y}_p^s$ are identified and collected into one single unknown vector $\underline{X}$. Thus, the non-linear problem reads:

$$\underline{R}(\underline{X}, \omega) = \underline{0} \text{ with: } \underline{R}(\underline{X}, \omega) = -\omega\mathbb{M}(\underline{X}) + \mathbb{C} + \mathbb{L}(\underline{X}) + \mathbb{Q}(\underline{X}, \underline{X})$$

where $\mathbb{M}(\underline{X}), \mathbb{C}, \mathbb{L}(\underline{X})$ and $\mathbb{Q}(\underline{X}, \underline{X})$ stand for, respectively, the mass operator, the constant external forcing, the linear operator and the quadratic one. The convenient quadratic expression of the residual $\underline{R}$ allows to solve the problem using the ANM in order to continue solution $\underline{X}$ in respect to the parameter ω. The main advantage of this approach is to compute exactly the associated Jacobian matrix of the residual $\underline{R}$. So, unknowns $\underline{X}$ and ω are sought as truncated series of the path-parameter a:

$$\underline{X} = \sum_{k=1}^{N} a^k \underline{X}_k \quad \text{and} \quad \omega = \sum_{k=1}^{N} a^k \omega_k \quad \text{with} \quad a = \langle X - X_0, X_1\rangle + (\omega - \omega_0)\omega_1$$

Then, the validity domain of one ANM step is evaluated according to: $a_{\max} = (\epsilon\|\underline{X}_1\|/\|\underline{X}_N\|)^{1/(N-1)}$ and a new continuation step can be performed: $\underline{X}_0^{\text{new}} = \underline{X}(a_{\max})$ and $\omega_0^{\text{new}} = \omega(a_{\max})$. More details can be found, for example, in [4, 5, 6, 7, 8].
This procedure is applied to analyse the dynamic response of the LO and the NES. The response curves are in good agreement with the numerical ones established in [1, 9] for a non-linear vibration absorber without electro-magneto-mechanical coupling (Fig.2).
Additional simulation are performed, for different values of the force F, to identify the appropriate values of the ANM parameters, mainly the number of harmonics H, the truncation order N and the tolerance threshold ϵ (Fig.3).

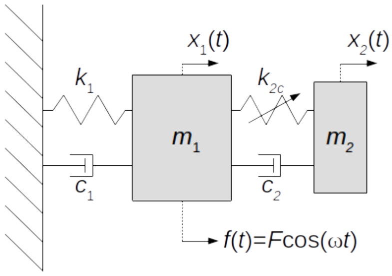

Figure 1: Single-degree-of-freedom oscillator non-linearly coupled with a NES

In particular, continuous solutions are obtained and several dynamic behaviours are highlighted according to the excitation force level F. Bifurcations (Hopf and Neimark-Sacker) are also computed in accordance with theoretical predictions obtained through a combination of the Complexification-Averaging method and the Multiple Scales method [1, 9].
The very first results show the relevancy of the numerical approach which is going to be used in order to study the dynamic behaviour of composite hydrofoil coupled to a NES [10].

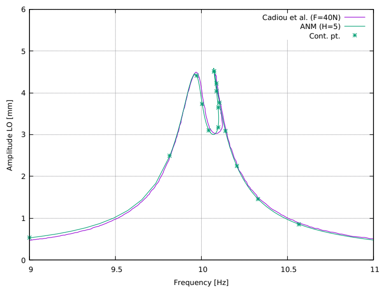

Figure 2: Evolution of the LO amplitude as a function of the frequency for $F = 40$ N with $H = 5$ harmonics. Comparison with result from [1].

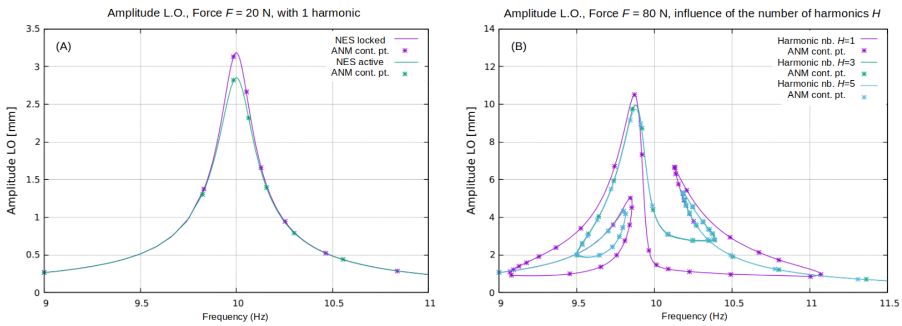

Figure 3: Evolution of the LO amplitude as a function of the frequency. Influence of the parameters on the dynamic behavior: $F = 20$ N with $H = 1$ harmonic (A) ; for $F = 80$ N with $H = 1, 3, 5$ harmonics (B).

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